

Spectral and Energy Efficiency Optimization for 5G Lens Array Based Millimeter-Wave Massive MIMO Systems

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Abstract-Lens-based beamspace mmWave MIMO is proposed for the future 5G wireless systems which can significantly decrease the number of power-dissipating radiofrequency (RF) chains required for channel links without perceptible system performance deterioration. The leakage of beam-power is a considerable issue because the angle-ofdepartures (AoDs) of channel paths cannot accurately map on the spatial sample points, which results in leakage of one beam power of path into other beams. This consequently degrades the sum-rate and energy efficiency of the system. To overcome this problem, we proposed a novel approach of deploying energyefficient phase-shifter network (PSN), gain-rotation (GR) and zero-forcing (ZF) algorithms to optimize the spectral efficiency (SE) and energy efficiency (EE) in mmWave Massive MIMO system based on lens-array antennas. The benefit of PSN is that it uses phase-shifters (PSs) which can select multiple beams by using a single RF-chain, whereas in conventional precoder, each RF-chain can select only one beam. The GR-algorithm is proposed in addition to PSN which maximize each user SNR by rotating the channel-gains. Simulation results shows that the proposed algorithms perform much better by improving the system spectral and energy efficiency as compare to the conventional algorithms. The results also validate our proposed analytical formulation.

Index Terms-mmWave, PSN, GR, ZF and RF Chain

I. INTRODUCTION

THE combination of mmWave and Massive MIMO technology is considered to be the benchmark technology for the future 5G wireless communication networks, because of its prominent features such as larger bandwidth (BW) that results in providing high data-rate transmission, enhanced energy efficiency (EE), and suitable for microcell and picocell cellular networks [1]. Due to such attractive qualities, it got the attention of many research and industrial communities. The rapid increase in the deployment of smartphones, tablets and notebooks requires more mobile data traffic which is a challenging problem for the service providers and the concerned researchers. According to the survey which stated that, new devices such as IoT modules and cloud sensors will surpass the existing devices and reach to 50 billion [2]. Moreover, the existing fourth-generation (4G) technology is not able to fulfill the demand of 1000times increase in data traffic for future 5G wireless mobile networks. To solve this issue, the concept of millimeter-wave (mmWave) massive MIMO is proposed which can effectively fulfill the consumers' demands.

The mmWave spectrum is from 30-300GHz which is underutilized, and this spectrum is utilized by incorporating the massive MIMO technology which makes the overall system performance much better. The issue of concern in the proposed scheme is that, each antenna in the system require its own dedicated RF-chain which is directly proportional to the number of used antennas. This causes more energy consumption and budget requirement. This consequently degrades the system throughput and energy-efficiency.

To decrease the number of power-dissipating RF-chains, the idea of beamspace MIMO is proposed [3]. The principle of such technique is that it uses lens-antenna array which concentrates the energy of different channel beams on different antennas, which is depicted in Fig. 1 (a). It basically performs a transformation of the conventional spatial-channel into the beamspace channel, which is shown in Fig. 1(b).



Fig. 1: Beamspace MIMO: (a) Lens-antenna array for energy concentration, (b) spatial vs beamspace channels

The beam-selection technique is used which reduces the number of RF-chains required, due to the fact that the mmWave channels characteristics are limited and scattering, thus the beamspace channel exhibit sparse-structure, which is the basis for RF-chain reduction.

The existing research literature does not work on the issue of power leakage in beamspace MIMO system which considerably degrades the system performance. It is due to the fact that, the lens-antenna array has fixed spatial sample points (SSPs), whereas the associated angle of departures (AoDs) of different channel paths are distributed continuously. This means that the channel paths AoDs cannot accurately map on the SSPs, resulting in the unavoidable power leakage of one beam into various other beams respectively. The [4] and [5] are based on conventional precoding for beamspace MIMO in which each beam is selected for a single path, which makes a one-to-one correspondence and thus, small part of the channel power can be collected that consequently implies considerable SNR loss. A simple but undesirable clue to this problem is to deploy more number of RF-chains which can receive the single channel path leaked power. This method can definitely eradicate the leakage problem but, it requires huge number of RF-chains as each antenna in mmWave Massive MIMO system requires its own RF-chain, which drastically increases the energy consumption requirement and complexity of the system.

In this paper, we proposed a novel approach of deploying energy-efficient phase shifter network (PSN), gain-rotation (GR) and zero-forcing (ZF) algorithms to overcome the power-leakage issue and increase the system spectral and energy-efficiency. The main difference of the proposed and the conventional algorithms is that, the proposed PSN uses phase-shifters (PSs) which can select multiple-beams for a single RF-chain which enables the reception of majority of the leaked beam-power, whereas the conventional scheme can only select a single beam for each RF-chain. The GRalgorithm is proposed in addition to PSN in order to deal with the constant-modulus bound of the non-convex on the RFdomain precoder, which is designed to boost each user SNR by rotating the selected beams channel-gains to the same direction. The ZF precoding is also proposed to improve the sum-rate and energy-efficiency with in terms of SNR of the proposed system in addition to PSN. Simulation results show that the proposed algorithms can efficiently optimize the power-leakage issue of the mmWave beamspace MIMO to attain the near-optimal sum-rate and energy-efficiency (EE) respectively. The results also depict that the proposed schemes perform much better than the conventional precoding schemes.

The remaining paper is organized as follows. Section II describes the proposed system model which is explained with analytical formulation. The simulation results are provided in Section III. Section IV concludes the paper.

II. SYSTEM MODEL

A typical system of mmWave massive MIMO is considered with N number of BS-antennas, K number of single antenna users and N_{RF} number of RF-chains respectively. The mmWave frequency is used for operation. The received signal-vector for all *K*-users can be expressed by:

$$y = \hat{H}^H x + n \tag{1}$$

Where,

y: is the received signal-vector,

 $\hat{H} = [\hat{h}_1, \hat{h}_2, \hat{h}_3, ..., \hat{h}_K] \in \mathbb{C}^{N \times K}$: is the beamspace channelmatrix,

 $\hat{h}_{K} \in \mathbb{C}^{N \times 1}$: is the beamspace channel-vector between the BS and *k*th-user,

 $x \in \mathbb{C}^{Nx1}$: is the transmitted signal-vector at the BS for all *K*-users,

 $n \in \mathbb{C}^{K_{x1}}$: is the noise-vector which follows the complex Gaussian distribution $CN(0, \sigma^2 I_K)$, where σ^2 is the noise-power.

For the power-constraint criterion, we follow the condition:

$$\|x\| \le P_T \tag{2}$$

Where,

 P_T : is the total transmit power.

The lens-antenna array acts as a unitary spatial discrete Fourier transform (DFT) matrix of order NxN, the rows of which consists of the *N*-orthogonal steering-vector. It is expressed by:

$$U = [a(\hat{\Phi}_1), a(\hat{\Phi}_2), a(\hat{\Phi}_3), \dots, a(\hat{\Phi}_N)]^H$$
(3)

 $a(\Phi) \in \mathbb{C}^{N\times 1}$: is the array steering-vector for the spatialdirection Φ which is obtained from:

$$\widehat{\Phi}_i = \frac{1}{\sqrt{N}} \left[i - \frac{N+1}{2} \right] \tag{4}$$

Where,

Where,

i = 0, 1, 2, ..., N: is the set of indices which cover the complete space.

The sparse beamspace and spatial channel matrix relationship can be expressed by:

$$\hat{H} = UH = [Uh_1, Uh_2, Uh_3, \dots, Uh_K]$$
 (5)

Where,

 $h_k \in \mathbb{C}^{N \times 1}$: is the kth-user spatial channel-vector.

A) mmWave Spatial Channel Model

The Saleh Valenzuela channel-model is deployed for the proposed mmWave clustered channel-model which is mostly used for this purpose [4]. The conventional spatial-domain channel is then expressed by:

$$h_{k} = \sqrt{\frac{N\mu_{k}}{N_{cl}^{k}N_{p}^{(k,l)}}} \sum_{l=0}^{N_{cl}^{k}} \sum_{l=0}^{N_{p}^{(k,l)}} \beta_{k,l}^{i} a(\Phi_{k,l}^{i})$$
(6)

Where,

 μ_k : is the large-scale fading factor of the *k*th-user,

 N_{cl}^k : is the number of clusters for the *k*th-user,

 $N_p^{(k,l)}$: is the number of paths within the *l*th-cluster of the *k*th-user,

 $\beta_{k,l}^{i}$: is the complex gain for the *i*th-path in the *l*th-cluster of the *k*th-user,

 $\Phi_{k,l}^{l}$: is the angle-of-departure (AoD) for the *i*th-path in the *l*th-cluster of the *k*th-user, which is distributed within a certain cluster for all *i* as $[\Phi_{k,l} - \frac{\tau_{k,l}}{2}, \Phi_{k,l} + \frac{\tau_{k,l}}{2}]$, where $\Phi_{k,l}$ is the average angular spread and $\tau_{k,l}$ is the average AoD in the *l*th-cluster of the *k*th-user respectively.

The steering-vector can be obtained by considering the typical N-antenna elements uniform linear array (ULA) as follows:

$$a(\Phi) = \frac{1}{\sqrt{N}} \left[e^{-j2\pi\Phi m} \right]_{m \in I(N)} \tag{7}$$

Where,

 $I(N) = [l - \frac{N-1}{2}, l = 0, 1, 2, ..., N - 1]$: is the set of indices of symmetric-set centered around-zero,

 Φ : is the spatial-direction which is obtained by

$$\Phi = \frac{d}{\lambda}\sin\theta \tag{8}$$

Where,

 θ : is the signal physical direction,

 λ : is the wavelength of the signal,

d: is the separation or spacing of antenna elements which satisfy the half-wave condition $(d = \frac{\lambda}{2})$ at the proposed mmWave frequencies.

B) Power Leakage Problem

The power leakage in beamspace MIMO occurs due to the fact that the SSPs of the lens-antenna array are fixed, whereas the channel path actual AoDs are continuously distributed. Thus, the path AoD of one channel cannot accurately map with the SSPs. Which results the power leakage of one channel beam onto multiple channel beams, that causes performance deterioration. Moreover, in the same cluster, the multiple paths power leakage is overlaid on the narrow range of $[\Phi_{k,l} - \frac{\tau_{k,l}}{2}, \Phi_{k,l} + \frac{\tau_{k,l}}{2}]$, which intensifies the power leakage problem further. This phenomenon is depicted in Fig. 2. In Fig. 2(a), there is no power leakage because there is an accurate mapping of the AoD and the lens SSPs. Fig. 2(b) shows the worst power leakage where the path AoD is in the middle of two SSPs. Fig. 2(c) shows the slight power leakage, where a slight mismatch occurs between the path AoD and the SSP. To determine the power-leakage quantity, we consider the single path scenario where, $N_{cl}^k = 1$ and $N_p^{(k,l)} = 1$ along with the worst power leakage of Fig. 2(b). If only one beam is selected having the highest power, then the ratio of the leaked-power to the total-power is given by:

$$\eta = 1 - \frac{1}{2\sum_{i=1}^{N} \frac{\sin^2(\frac{\pi}{2N})}{\sin^2((2i-1)\frac{\pi}{2N})}}$$
(9)

It is clear from (9) that for the considered single path scenario, the power leakage severe. Which means that there is a considerable amount of SNR loss that require optimization.

This ratio tells us that how much power will be leaked from the total power of the beam. To understand it further, consider N=256 in (9) which results about 59.5% of power-leakage.



Fig. 2: Power leakage issue: (a) No power leakage, (b)worst power-leakage, (c) Slight power leakage

C) PSN Precoding

We illustrate three different precoding methods for the beamspace MIMO system. It is depicted in Fig. 3. The powerleakage issue is not considered in Fig. 3(a) and only singlebeam (SB) is selected through single RF-chain for single-user. It is called SB-precoding and the power-consumption of such scheme is given by:

$$P_{SB} = P_T + P_{BB} + P_{RF}K + NKP_{SW}$$
(10)

Where,

 P_{BB} : is the baseband power, P_{RF} : is the RF-chain power, P_{SW} : is the switch power.

This precoding structure deploys small number of RFchains which consume lower energy. But as discussed in (9) that the SB precoding has considerable SNR loss which is due to the worst power leakage issue. A simple but undesired clue to this problem is to deploy much more RF-chains to receive more beams power which is illustrated in Fig. 3(b).



Fig. 3: Beamspace MIMO precoding: (a) single beam conventional precoding, (b) multiple beam with multiple RF-chains precoding, (c) proposed PSN precoding

This precoding structure is called multiple beam through multiple RF-chains precoding or in short, MBMRF-precoding structure. The MBMRF power consumption can be expressed by:

 $P_{MBMRF} = P_T + P_{BB} + P_{RF}B_T + NP_{SW}B_T$ (11) Where, $B_T = \sum_k B_k$: is the total number of selected-beams, B_k : is the number of selected-beams for the *k*th-user,

This precoding structure can collect the leaked-power, but the drawback is that, it requires lot of RF-chains. To understand it further, let us consider 40 users (*K*) and the total of number of selected beams of 4 (B_T), therefore, the required number of RF-chains will be 4x40=160 which means higher energy consumption, more cost and higher hardware complexity respectively. To overcome this issue, we propose the PSN precoding as shown in Fig. 3(c) to solve the powerleakage problem. The main idea of this algorithm is that, a single RF-chain can select multiple-beams by using analog-PSN. More specifically, a single RF-chain is connected to an arbitrary subset of the total number of antennas via a switch network. Moreover, a single antenna can be connected to one RF-chain at most. Also, the phase-shifter (PS) is present on each antenna to rotate its signals. It is important to note that the total number of PSs is N which is not a huge number. The proposed PSN precoding power consumption can be represented by:

$$P_{PSN} = P_T + P_{BB} + P_{RF}K + P_{SW}NK + P_{PS}B_T$$
(12)
Where,

 P_{PS} : is the phase-shifter power consumption.

The transmitted signal for the proposed PSN algorithm is given by:

$$x = P_{RF} P_{BB} S \tag{13}$$

Where,

$$P_{RF} = \left[P_{RF}^{[1]}, P_{RF}^{[2]}, P_{RF}^{[3]}, \dots, P_{RF}^{[N_{RF}]} \right] \in \mathbb{C}^{N \times N_{RF}}: \text{ is the RF-precoder,}$$

 $P_{BB} = \left[P_{BB}^{[1]}, P_{BB}^{[2]}, P_{BB}^{[3]}, \dots, P_{BB}^{[N_{RF}]} \right] \in \mathbb{C}^{N_{RF}xK}$: is the baseband-precoder,

 $S \in \mathbb{C}^{K \times 1}$: is the signal-vector of the source.

Specifically, as P_{RF} is practically realized via the PSs and switches, therefore, it should satisfy the following spatial hardware constraint:

$$\left| \begin{bmatrix} P_{RF}^{[1]} \end{bmatrix}_{j} \right| = \begin{cases} \frac{1}{\sqrt{B_{i}}}, & j \in B_{i} \\ 0, & otherwise \end{cases}$$
(14)

Where,

 B_i : is the selected beams set of indices for the *i*th-user.

Due to such constraint of (14) for the hardware, the conventional algorithms for the beamspace MIMO are hard to be deployed for the proposed PSN algorithm. For this purpose, we proposed an additional gain-rotation (GR) algorithm which is discussed below.

D) GR Precoding

This precoding provides rotation of channel path gains of the selected beams to the same direction which increases each user SNR for optimum performance. The equivalent channel matrix for the GR-based precoding between the users and the RF-chains is given by:

$$\breve{H} = P_{RF}^{H} \widehat{H} = [\breve{h}_{1}, \breve{h}_{2}, \breve{h}_{3}, \dots, \breve{h}_{K}]$$
(15)

We assume that, the channel state information (CSI) of all users is present at the base station (BS), which is typically estimated by the CS-algorithms with low pilot-overhead. The power constraint condition for the per-user case of the proposed GR-algorithm is given by:

$$\left\| P_{RF} P_{BB}^{[k]} \right\|^2 \le \frac{P_T}{\kappa}, \text{ for all } k \tag{16}$$

As the large number of BS can offer optimum spatial resolution, and the number of dominant clusters are limited for each user, it can be assumed that the average path AoDs are sufficiently separated from each other for each user. Therefore, the inter-user interference (IUI) is not severe in the beamspace MIMO systems, which is its distinguish property [6]. Thus, the optimum precoder for the proposed algorithm is then represented by:

$$\left[P_{RF}^{opt}, P_{BB}^{opt}\right] = \frac{arg \max}{P_{RF}} \sum_{BB}^{K} \sum_{k=1}^{K} \left[\breve{h}_{k}^{H} P_{BB}^{k}\right]^{2}$$
(17)

The baseband precoder in (17) has the optimal precoder solution of matched filter (MF) which is given by:

$$P_{BB}^{k} = \alpha_{k} \check{h}_{k}^{H} \tag{18}$$

Where,

 α_k : is the transmit power normalization factor for the *k*th-user. Now, putting (18) into (17), we get the P_{RF} precoder as follows:

$$P_{RF}^{opt} = \frac{\arg \max}{P_{RF}} \sum_{k=1}^{K} \left[\alpha_k \breve{h}_k^H \breve{h}_k \right]^2 \tag{19}$$

This equation is obtained for the purpose of eliminating the transmit power constraint of (16). It is due to the α_k factor which can be adjusted to get rid of such constraint. Now, the IUI factor elimination is required, for which we use the following illustration in Fig. 4.



Fig. 4: GR-based channel gains rotation: (a) combination without rotation, (b) proposed combination with rotation

Using the above illustration, (19) can be expanded for the objective function as follows:



The IUI term is eliminated due to the distinguish property of the proposed beamspace MIMO which is discussed above. The combination of the selected beams can be elaborated more by considering that, the non-zero element positions in the RF-precoder are known after beam-selection. Therefore, maximization of (20) means the rotation of the concerned beams to same direction, which is similar to vector-addition process and is given by:

$$\frac{[P_{RF}^{k}]_{e_{1}}}{[P_{RF}^{k}]_{e_{2}}} = \frac{\left|\frac{(\hat{h}_{k})_{e_{1}}}{(\hat{h}_{k})_{e_{2}}}\right|}{\left|\frac{(\hat{h}_{k})_{e_{1}}}{(\hat{h}_{k})_{e_{2}}}\right|}$$
(21)

Where,

 $e_1, e_2 \in B_k$: are the reference elements. This combination process is clearly understood from Fig. 4 where both channel gains are rotated to maximize the combined value. The greedy beam selection scheme is used to collect the leaked power. Its main idea to consider the strongest power beam to position the cluster, then select the subsequent beams with highest power leakage in a greedily manner. The flowchart describes the overall proposed algorithm in Fig. 5.



Fig. 5: Proposed PSN GR-algorithm for power-leakage optimization

Table I shows the detail of the simulation parameters of PSN GR-precoding for the proposed system which is utilized for simulations results.

TABLE I: Proposed system parameters for	PSN	GR-algorithm
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S. No	Parameter Sy		Value
1	Number of lens antennas (beams) at the BS	Ν	520
2	Number of users	К	10
3	System Bandwidth	BW	500MHz
4	Noise power spectral density	PSD	-174dBm/Hz
5	Number of antenna per user	N _{user}	1
6	Number of clusters	N ^k _{cl}	1
7	Number of paths in each cluster	$N_p^{(k,l)}$	10
8	Transmit power	P _T	20 dBm
9	Number of RF chains per user	N _{RF}	1
10	Baseband power consumption	P_{BB}	23.01dBm
11	RF power consumption	P _{RF}	23.802dBm
12	Switch power consumption	P _{SW}	6.98dBm
13	Phase-shifter power consumption	P _{PS}	14.77dBm
14	Distance between BS and users	d	10m

E) ZF Precoding

The zero-forcing precoder for the proposed PSN-algorithm is also based on the matched-filter (MF) for determining the system performance evaluation. It is then connected to subconnected structure of hybrid analog precoding to reduce the number of RF-chains required. Table II shows the detail of the simulation parameters of ZF-precoding for the proposed system which is utilized for simulations results.

S. No	Parameter	Symbol	Value
1	Number of lens antennas (beams) at the BS	N	520
2	Number of user antenna	Κ	1
3	Number of clusters	N_{cl}^{k}	1
4	Number of paths in each cluster	$N_p^{(k,l)}$	100
5	Number of RF chains per user	N _{RF}	1
6	Signal-to-Noise Ratio	SNR	20dB

TABLE II:	Proposed	System	parameters	for PSN	ZF-algorithm
	1	~			6

III. SIMULATION RESULTS

Simulation results are given to verify our analytical formulation. Fig. 6 depicts the sum-rate (SE) comparison of the proposed PSN-algorithm and the conventional algorithms against the number of channel beams. It is worth notable that, the single beam (sRF) precoding always select only one beam for each user, whereas the proposed PSN-algorithm and the FD/MBMRF algorithm uses multiple beams for each user, which take changes in these algorithms.



Fig. 6: Comparison of the proposed PSN-algorithm with conventional algorithms in terms of sum-rate

It is clear from the figure that our proposed PSN-algorithm achieves much higher sum-rate as compare to the single RF per user (sRF/user) algorithm. It is also evident that, the sumrate gap between the proposed PSN and the conventional sRF algorithm gets larger when the number of beams increases. Furthermore, our proposed precoding scheme also achieve the near-optimal performance of the fully-digital (FD) precoding which means that this scheme has the ability to effectively optimize the power-leakage issue in the proposed mmWave beamspace massive MIMO systems.



Fig. 7: Comparison of the proposed PSN-algorithm with conventional algorithms in terms of energy efficiency

Fig. 7 compares the energy efficiency (EE) of the proposed PSN-algorithm with the conventional precoding schemes against the number of beams. It is important to note that as, the FD/MBMRF scheme has better sum-rate in Fig. 6, but it has poor energy-efficiency (EE) in Fig. 7, that is also lower than the sRF precoding, which is not desirable. It is due to the fact that the FD/MBMRF precoding deploys huge number of power-consuming RF-chains. On the other hand, our proposed PSN-precoding shows the best energy efficiency performance than the conventional schemes. It is because the proposed precoding scheme deploys small number of phaseshifters (PSs) for leaked power collection that consume lowenergy. Therefore, it is concluded that, the proposed schemes are suitable for both spectral and energy efficiency optimization. Fig. 8 compares the sum-rate of the proposed PSN ZF-precoding with the conventional precoding schemes against the signal-to-noise ratio (SNR). It is clear from the figure that, the proposed PSN ZF-algorithm achieves better sum-rate performance than the single beam (sRF) precoding, which is also close to the near-optimal FD-precoding.



Fig. 8: Comparison of the proposed PSN ZF-algorithm with conventional algorithms in terms of sum-rate



Fig. 9: Comparison of the proposed PSN ZF-algorithm with conventional algorithm in terms of energy efficiency

Moreover, the sum-rate gap between the proposed precoding scheme and the sRF precoding increases with increasing SNR values, which means that, our proposed scheme can perform better than the conventional schemes in higher SNR regime. Fig. 9 illustrates the comparison of the proposed PSN ZF-precoding with the ideal precoding in terms of energy efficiency.

It is clear that, the proposed scheme can achieve close performance with the ideal precoding scheme which makes it suitable from practical system perspective.

IV. CONCLUSION

In this paper, we investigate the power-leakage issue and the number of RF-chains required in the mmWave massive beamspace MIMO systems. The PSN, GR and ZF precoding schemes are proposed to optimize the concerned problem. Its main idea is to deploy single RF which can select multiple beams through multiple phase-shifters (PSs) for collection of the majority of the leaked power of the beamspace channel. The main aim of GR algorithm is to maximize each user SNR by rotating the selected users beams channel-gains to the same direction. The ZF precoding is based on MF, is used in addition to PSN to analyze the system performance in terms of SNR. Simulation results show that, our proposed algorithms are able to effectively optimize the concerned power-leakage issue in mmWave beamspace massive MIMO systems. The power leakage optimization also increases the overall system performance, which is evident from the results that the sum-rate (SE) and EE are improved by deploying the proposed precoding schemes. This research work can further be extended by considering multiple user-antennas at the userterminal (UT) and deploy such algorithms with different channel models.

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