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Robust Transmit Beamforming Based on Probability Specification in Cellular Cognitive Radio Networks

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***Abstract*– In this paper, we design a downlink beamforming vector for a multiuser MISO cellular cognitive radio network with imperfect channel state information (CSI) at the secondary base station. The idea is to minimize the secondary BS transmit power subject to outage probability constraints to the QOS of the secondary users (SUs) and the interference power to the primary users. This led to a non-convex optimization problem but was reformulated to a convex-one using semi definite relaxation technique. The resulting problem can be solved efficiently using interior point methods. Computer simulations shows that the probabilistic approach used is more power efficient than the less flexible worst-case approach.**

***Index Terms*– Beamforming, Cognitive Radio and Secondary User**

I. INTRODUCTION

THE increasing demand for wireless services has urged researchers to seek an efficient way of utilizing the available radio spectrum. Wireless communication has been expanding at a fast pace and this has lead to an increase in demand for the spectrum. Currently, research is focused on utilising the spectrum as efficiently as possible. Cognitive radio (CR) is a promising solution to solve the spectrum scarcity problem [1]. Cognitive radio is an adaptive, intelligent radio and network technology that can detect available channels in a wireless spectrum and changes transmission parameters enabling more communication to run concurrently. Cognitive radio network (CR-Net) architecture usually comprises of the secondary users (SUs) which are operating in an unlicensed band or coexisting with primary users (PUs) in the same licensed band. The PUs are the original licensed users to operate in a particular spectrum band while the SUs are the unlicensed users and they attempt to use the spectrum opportunistically. In practice, the secondary spectrum usage is only possible if the SUs can cause an acceptably small performance degradation to the PUs. A situation whereby PUs share the same spectrum with the SUs is known as underlay cognitive Network, provided that the amount of interference power (IP) to each PU receiver is kept below a certain threshold, whereas if the SUs utilize the spectrum when the primary user is not using it, is called overlay cognitive network [2]. In this paper, we are interested in the underlay cognitive radio network.

Fig. 1 illustrate the downlink scenario of a single cell multiuser multiple-input-single-output (MISO) CR-Net with K SUs and coexisting with M PUs.

In Fig. 1 there are two sets of channel-state information (CSI) that we are considering for the system design. The first set is the channel between the SU-Transmitter (SU-TX) and the SU-Receivers (SU-RXs) which we term the SU-link CSI while the other set is the channel between the SU-TX and the PU-Receivers (PU-RXs) which we term the PU-link CSI. In this paper we regard the interfering transmission power from the PU-Transmitter to the cognitive radio network as part of the noise term. For a good transmission design in a downlink channel, the knowledge of the downward link CSI is needed. This knowledge is usually acquired by transmitting pilot symbols from the downlink transmitters to the downlink receivers; the feedback of the estimated CSI is also transmitted from the receivers back to the transmitters. For cognitive radio network (CRN) in an underlay setting to operate without too much interference to the PUs, downlink Beamforming must play a vital role. The performance improvements brought by the use of downlink Beamforming can be realized if accurate channel state information is available at the transmitter. Note that from a mathematical

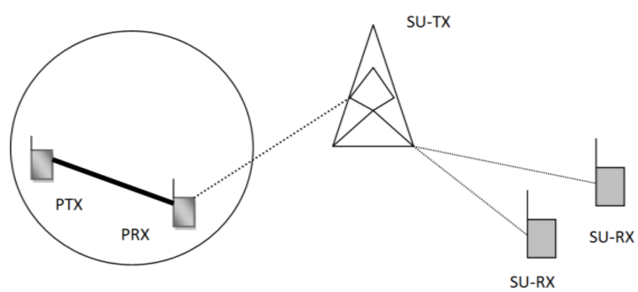

Fig. 1: A single-cell underlay CR-Net

point of view, the CR Beamforming can be viewed as a conventional Beamforming problem with additional constraints on the interference to the PUS.

II. RELATED WORKS

Several Beamforming techniques have been developed assuming perfect CSI in cellular systems. In [3], the availability of instantaneous CSI at the transmitter is considered while the technique in [4] relies on perfect covariance-based CSI at the transmitter. In practice, however, because of the time varying nature of wireless channels, it is impossible to perfectly acquire the CSI due to channel variations or feedback errors. In this paper, we will consider the transmit design for a multiuser MISO cellular CR-Net with uncertain CSI in both the SU-link and the PU-link using probabilistic approach and compare it with the worst case design method.

Several related works have been developed which assume perfect CSI for CR-net beamforming techniques, see [6]-[7]. Methods considering erroneous CSI are considered in [8]-[9]. Authors in [8] used several approximations which are conservative modifications of the QOS and PU interference constraints, which are termed the worst case approaches. Note however, that in wireless communications, it is not practical to use deterministic upper bounds on the norms of the channel errors. As the wireless channel varies randomly, it is more logical to exploit the statistical nature of these errors. This motivates us to consider a probabilistic model for the mismatches.

In this paper we propose a robust approach to cellular cognitive downlink beamforming where the secondary base station transmit power is optimized, subject to the outage probability constraints. Note, several outage probability-based design techniques have been earlier proposed for power control in single-antenna systems [10]-[11], multi-antenna system [12], cognitive network [13], cellular network [14] but apply it to a different problem altogether. In [13] the outage probability based problem was regarded to be a mathematical equivalent to the worst-case formulation; however the authors do not actually utilize it for further analysis or comparison. In this paper we utilize the statistical outage probability based problem as a basis for comparison with a conventional based worst-case formulation which has deterministic upper bounds on the norms of the channel errors; and results from our simulation shows that the outage probability based approach out performs that of the worst-case design based robust transmit beamforming technique.

Notations: Matrices and vectors are type faced using bold uppercase and lower case letters, respectively. The transpose and conjugate transpose of the matrix A are denoted as A^T and A^H, respectively. The trace of a matrix is annotated using Tr[·]. The PSD of the matrix A is depicted using A≥0. The symbol ≜ means "defined as" C^{mxn} is used to describe the complex Gaussian random variable with the variance of σ^2 is denoted using $\mathcal{CN}(0, \sigma^2)$, for a vector x, ||x|| is the Euclidean norm, where as the norm of a matrix like $\|A\|$ is the frobenius norm.

III. PROBLEM STATEMENT

Consider a single-cell CR-net coexisting with a single-cell PR-net with M PUs and K SUs. It is assumed that the base station of the secondary user is equipped with N antennas while both the primary and the secondary users consist of only a single antenna. The signal transmitted by the secondary user base station is:

$$\mathbf{x}(t)=\sum_{k=1}^{K} \boldsymbol{s}_k = \boldsymbol{WS} \quad (1)$$

where $\mathbf{S}=[s_1,......,s_k]^T \in C^{kX1}$ contains the transmitted symbols and $\mathbf{W}=[\boldsymbol{w}_1,.....,\boldsymbol{w}_k] \in C^{NXK}$ is called the pre-coding matrix; w_k is the beamforming weight vector for the *k*th secondary user and is defined as $\mathbf{w}_k \in C^{Nx1}$. The received signal at the secondary *k*th user terminal can be written as:

$$r_k(t)=\boldsymbol{h}_k^H \boldsymbol{x}(t) + n_k(t) \quad (2)$$

The $\{\boldsymbol{h}_k \in C^{NX1}\}_{k=1}^{K}$ is the channel from SU-TX to each SU-RX. $n_k(t)$ is the zero mean circularly symmetric AWGN component with variance σ_k^2. Inserting (1) into (2) and applying the statistical expectation E{·} over the random channel signal and noise realizations, we obtain that the received signal power $p_{r,k} = E\{|r_k(t)|^2\}$ of the kth SU can be expressed as:

$$p_{r,k} = \boldsymbol{w}_k^H \boldsymbol{R}_k \boldsymbol{w}_k + \sum_{\substack{i=1 \\ i\neq k}}^{k} \boldsymbol{w}_i^H \boldsymbol{R}_k \boldsymbol{w}_i + \sigma_k^2 \quad (3)$$

where $\boldsymbol{R}_k \triangleq E\{\boldsymbol{h}_k \boldsymbol{h}_k^H\}$ is the downlink channel covariance matrix for the *k*th SU. Also it is assumed $E\{s_k(t)\} = 0$ and $E\{|s_k(t)|^2\} = 1$ for all k=1,.....,K.

The *SINR* at the *k*th SU-RX $SINR_k$ is given by:

$$SINR_k \triangleq \frac{\boldsymbol{w}_k^H \boldsymbol{R}_k \boldsymbol{w}_k}{\sum_{\substack{i=1 \\ i\neq k}}^{k} \boldsymbol{w}_i^H \boldsymbol{R}_k \boldsymbol{w}_i + \sigma_k^2} \quad (4)$$

in addition, the received, signal at the *m*th PU is given as:

$$Z_m = \sum_{k=1}^{K} \boldsymbol{g}_m^H \boldsymbol{w}_k s_k + v_m \quad (5)$$

where $\{\boldsymbol{g}_m \in C^{NX1}\}_{m=1}^{M}$ is the channel from SU-TX to each PU-RX. v_m is the received noise. The interference power IP_m to this PU-RX is given as:

$$IP_m \triangleq \sum_{k=1}^{K} \boldsymbol{w}_k^H G_m \boldsymbol{w}_k \quad (6)$$

where $\boldsymbol{G}_m \triangleq E\{\boldsymbol{g}_m \boldsymbol{g}_m^H\}$ is the downlink covariance matrix for the *m*th PU.

The design objective is usually to minimize the transmitted power while guaranteeing that the SINR at each SU-RX for all the channel realizations is higher than the QOS-constrained threshold $\{SINR_k \geq \gamma_k\}_{k=1}^{K}$ and simultaneously the IP at each PU-RX is less than the PR-Net imposed threshold $\{IP_m \leq k_m\}_{m=1}^{M}$. Mathematically the resulting optimization problem can be described as:

$$\min_{\{w_k\}_{k=1}^{K}} \quad \sum_{k=1}^{K} \|w_k\|^2$$

subject to $\frac{\boldsymbol{w}_k^H \boldsymbol{R}_k \boldsymbol{w}_k}{\sum_{i=1, i\neq k}^{K} \boldsymbol{w}_i^H \boldsymbol{R}_k \boldsymbol{w}_i + \sigma_k^2} \geq \gamma_k$ k=1,........K

$\sum_{k=1}^{K} \boldsymbol{w}_k^H G_m \boldsymbol{w}_k \leq k_m$ m=1,........M (7)

By introducing a new variable $\boldsymbol{W}_k = \boldsymbol{w}_k \boldsymbol{w}_k^H$ and using semidefinite relaxation, (7) which is non-convex problem can be converted into a semi definite constraint problem [5] as:

minimize $\sum_{k=1}^{K} Tr\{\boldsymbol{W}_k\}$ subject to

$Tr\{\boldsymbol{W}_k \boldsymbol{R}_k\} - \gamma_k \sum_{i=1, i\neq k}^{K} Tr\{\boldsymbol{W}_i \boldsymbol{R}_k\} \geq \gamma_k \sigma_k^2$
k=1,.......,K

$\sum_{k=1}^{K} Tr\{\boldsymbol{W}_k \boldsymbol{G}_m\} \leq k_m$
m=1,........M

$\boldsymbol{W}_k \geq 0$
k=1,.......,K (8)

A) Worst Case Performance Optimization

The transmitter beamforming design in (8) assumes perfect CSI at the base station. But for a robust algorithm assuming uncertainty in the CSI; the covariance matrix for the *k*th SU will become $\boldsymbol{R}_k + \boldsymbol{E}_k$, where as the covariance matrix for the *m*th PU will become $\boldsymbol{G}_m + \boldsymbol{E}_m$ where $\boldsymbol{R}_k \in C^{NXN}$, $\boldsymbol{G}_m \in C^{NXN}$ are the presumed downlink channel covariance matrix for the *K*th SU and the Mth Pu respectively and $\boldsymbol{E}_k \in C^{NXN}$ and $\boldsymbol{E}_m \in C^{NXN}$ are the error matrix that corresponds to the estimates errors in $\boldsymbol{R}_k$ and $\boldsymbol{G}_m$ respectively. Throughout this paper, we will regard $\boldsymbol{E}_k$ and $\boldsymbol{E}_m$ as uncertainty matrices because they characterizes the mismatches between the actual and presumes(estimated) downlink channel covariance matrices. Note that the following matrices; $\boldsymbol{R}_k, \boldsymbol{G}_m, \boldsymbol{E}_k$ and $\boldsymbol{E}_m$ are all hermitian.

The original problem is therefore modified as:

minimize $\sum_{k=1}^{K} Tr\{\boldsymbol{W}_k\}$ subject to

$Tr\{\boldsymbol{W}_k(\boldsymbol{R}_k + \boldsymbol{E}_k)\} - \gamma_k \sum_{i=1, i\neq k}^{K} Tr\{\boldsymbol{W}_i(\boldsymbol{R}_k + \boldsymbol{E}_k)\} \geq \gamma_k \sigma_k^2$
k=1,.......,K

$\sum_{k=1}^{K} Tr\{\boldsymbol{W}_k(\boldsymbol{G}_m + \boldsymbol{E}_m)\} \leq k_m$
m=1,........M

$\boldsymbol{W}_k \geq 0$
k=1,.......,K (9)

In the worst-case performance optimization, the beamformer weight vector should be designed using a set of constraints on the worst possible errors. In [10], the robust transmit beamforming technique is based on the conditions that the frobenius norms of the error matrices $\boldsymbol{E}_k$ and $\boldsymbol{E}_m$ are upper and lower bounded by a known constant and the problem considered is to minimize the transmit power subject to QOS constraint for the SUs and IP constraints for the PUs ; that should be satisfied for the worst -case matrices $\boldsymbol{E}_k$ *and* $\boldsymbol{E}_m$ which are bounded in their frobenius norm as $\|\boldsymbol{E}_k\| \leq \varepsilon_k$ and $\|\boldsymbol{E}_m\| \leq \varepsilon_m$ respectively where $\varepsilon_m > 0$ and $\varepsilon_k > 0$. The resulting problem will be modified as shown in [8] as:

minimize $\sum_{k=1}^{K} Tr\{\boldsymbol{W}_k\}$ subject to

$Tr\{\boldsymbol{W}_k(\boldsymbol{R}_k - \varepsilon_k I)\} - \gamma_k \sum_{i=1, i\neq k}^{K} Tr\{\boldsymbol{W}_i(\boldsymbol{R}_k + \varepsilon_k I)\} \geq \gamma_k \sigma_k^2$
k=1,.......,K

$\sum_{k=1}^{K} Tr\{\boldsymbol{W}_k(\boldsymbol{G}_m + \varepsilon_m I)\} \leq k_m$
m=1,........M

$\boldsymbol{W}_k \geq 0$
k=1,.......,K (10)

$\boldsymbol{W}_k = \boldsymbol{W}_k^H$
for k=1,.......K

Eq. (10) is a SDP problem; it can be straightforwardly solved using convex optimization algorithms like CVX. Unfortunately, this approach requires the norms of $\{\boldsymbol{E}_k\}_{k=1}^{K}$ and $\{\boldsymbol{E}_m\}_{m=1}^{M}$ to be bounded by ε_k and ε_m respectively. This stringent requirement may not be satisfied in practice, and moreover, the worst-case approach can be overly pessimistic because the probability of the actual worst-case errors may be extremely low [15]. Therefore, probabilistic (soft-constrained) design provide a more realistic and flexible alternative to the deterministic worst case design.

B) Outage Probability Approach

In this section, we formulate a probabilistic approach to robust downlink beamforming for cellular cognitive network based on the outage probability. The idea is to replace the SINR and the IP in the formulation of the worst-case based downlink beamformer by more flexible probabilistic constraints. The resulting problem can be described as [16].

$\min_{\boldsymbol{w}_k} \|\boldsymbol{w}_k\|^2$

subject to

$P_k \geq p_k$ for all k=1,....,K

$P_m \geq p_m$ for all m=1,.....,M (11)

$P_k = \Pr\,(SINR_k \geq \gamma_k)$

$$=Pr\left(\frac{\boldsymbol{W}_k^H(\boldsymbol{R}_k + \boldsymbol{E}_k)\boldsymbol{w}_k}{\sum_{i=1, i\neq k}^{K} w_i^H(\boldsymbol{R}_k + \boldsymbol{E}_K)\boldsymbol{W}_i + \sigma_k^2} \geq \gamma_k\right) \quad (12)$$

and $P_m = \Pr\,(IP_m \leq k_m)$

$$=\Pr\,(\textstyle\sum_{k=1}^{K} \boldsymbol{w}_k^H(\boldsymbol{G}_m + \boldsymbol{E}_m)\boldsymbol{w}_k \leq k_m) \quad (13)$$

where Pr (·) is the probability operator. P_k and P_m defines the probability that the *K*th and *M*th users are not in outage and p_k and p_m are preselected threshold value. The non-outage probabilities for the Kth and Mth users are defined as the probabilities of the SINR to be greater than the threshold value γ_k and the probability of the IP to be less than the threshold value k_m.

Note that the outage probabilities in eq. (11) is defined as 1-P_k and 1-P_m respectively. i.e; as the probability that the SINR of (7) is below the threshold value γ_k and that of the IP of (7) is greater than the threshold value of k_m.

We can express (12) and (13) as:

$$P_k = P_r\{Tr(\boldsymbol{W}_k(\boldsymbol{R}_k + \boldsymbol{E}_k)) \geq \gamma_k \sum_{\substack{i=1\\ i\neq k}}^{K} Tr\big(\boldsymbol{W}_i(\boldsymbol{R}_k + \boldsymbol{E}_k)\big) + \gamma_k\sigma_k^2\} \quad (14)$$

$$P_m = \Pr\{\sum_{k=1}^{K} Tr(\boldsymbol{W}_k(\boldsymbol{G}_m + \boldsymbol{E}_m)) \leq k_m\} \quad (15)$$

introducing the matrix $\boldsymbol{Z}_k$

$$\boldsymbol{Z}_k = \boldsymbol{W}_k - \gamma_k \sum_{\substack{i=1\\ i\neq k}}^{K} \boldsymbol{W}_i \quad (16)$$

(14) can now be rewritten as:

$$P_k = P_r\{Tr((\boldsymbol{R}_k + \boldsymbol{E}_k)\boldsymbol{Z}_k) \geq \gamma_k\sigma_k^2\} \quad (17)$$

simultaneously; introducing the auxiliary matrix $\boldsymbol{C}$

$$\boldsymbol{C} \triangleq \sum_{k=1}^{K} \boldsymbol{W}_k \quad (18)$$

Hence (15) can now be written as:

$$P_m = \Pr\{Tr((\boldsymbol{G}_m + \boldsymbol{E}_m)\boldsymbol{C}) \leq k_m\} \quad (19)$$

To obtain a mathematically tractable formulation, we will assume that real valued diagonal and complex valued upper or lower triangle elements of $\boldsymbol{E}_k$ and $\boldsymbol{E}_m$ are zero mean , independent Gaussian values with a variance of $\sigma_{e_k}^2$ and $\sigma_{e_m}^2$.The Gaussian part of this assumptions is motivated by the fact that the covariance matrix errors are typically caused by multiple independent "error sources".

Let us define our new real-valued random variables as:

$$y_k \triangleq Tr\{(\boldsymbol{R}_k + \boldsymbol{E}_k)\boldsymbol{Z}_k\} \quad (20)$$

$$y_m \triangleq Tr\{(\boldsymbol{G}_m + \boldsymbol{E}_m)\boldsymbol{C}\} \quad (21)$$

Then

$$E\{y_k\} = E\{Tr\big((\boldsymbol{R}_k + \boldsymbol{E}_k)\boldsymbol{Z}_k\big)\} = Tr\{\boldsymbol{R}_k\boldsymbol{Z}_k\} \quad (22)$$

$$E\{y_m\} = E\{Tr\big((\boldsymbol{G}_m + \boldsymbol{E}_m)\boldsymbol{C}\big)\} = Tr\{\boldsymbol{G}_m\boldsymbol{C}\} \quad (23)$$

To compute the variance of y_k , note that $\sigma_{e_k}^2 = E(y_k - E(y_k))^2$

$$= E(Tr\{(\boldsymbol{R}_k + \boldsymbol{E}_k)\boldsymbol{Z}_k\} - E(Tr\{(\boldsymbol{R}_k + \boldsymbol{E}_k)\boldsymbol{Z}_k\}))^2$$

$$= E(Tr\{(\boldsymbol{R}_k + \boldsymbol{E}_k)\boldsymbol{Z}_k\} - Tr\{\boldsymbol{R}_k\boldsymbol{Z}_k\})^2$$

$$= E(Tr\{\boldsymbol{R}_k\boldsymbol{Z}_k\} + Tr\{\boldsymbol{E}_k\boldsymbol{Z}_k\} - Tr\{\boldsymbol{R}_k\boldsymbol{Z}_k\})^2$$

$$= E\{Tr\{\boldsymbol{E}_k\boldsymbol{Z}_k\}Tr\{\boldsymbol{E}_k\boldsymbol{Z}_k\}^H\}$$

$$= E\{vec(\boldsymbol{Z}_k^H)^H vec(\boldsymbol{E}_k) vec(\boldsymbol{E}_k)^H vec(\boldsymbol{Z}_k^H)\}$$

$$= vec(\boldsymbol{Z}_k^H)^H E\{vec(\boldsymbol{E}_k) vec(\boldsymbol{E}_k)^H\} vec(\boldsymbol{Z}_k^H)$$

$$= \sigma_{e_k}^2\, vec(\boldsymbol{Z}_k^H)^H I_{NN}\, vec(\boldsymbol{Z}_k^H)$$

$$= \sigma_{e_k}^2 Tr(\boldsymbol{Z}_k^H\boldsymbol{Z}_k) = \sigma_{e_k}^2\|\boldsymbol{Z}_k\|^2 \quad (24)$$

Similarly, the variance of y_m can be computed as:

$$E\{Tr\{\boldsymbol{E}_m\boldsymbol{C}\}Tr\{\boldsymbol{E}_m\boldsymbol{C}\}^H\} = \sigma_{e_m}^2\|\boldsymbol{C}\|^2 \quad (25)$$

The non-outage probability $P_k = \Pr(y_k \geq \gamma_k\sigma_k^2)$ can be expressed as a probability density function.

$$P_k = \int_{\gamma_k\sigma_k^2}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_{e_k}\|\boldsymbol{Z}_k\|} exp\left(-\frac{(y_k-\mu_k)^2}{2\,\sigma_{e_k}^2\|\boldsymbol{Z}_k\|^2}\right)dy \quad (26)$$

where $\mu_k = Tr\{\boldsymbol{R}_k\boldsymbol{Z}_k\} \geq 0$

Similarly $P_m = \Pr(y_m \leq k_m)$ can be expressed as a probability density function.

$$P_m = \int_{-\infty}^{k_m} \frac{1}{\sqrt{2\pi}\,\sigma_{e_m}\|\boldsymbol{C}\|} exp\left(-\frac{(y_m-\mu_m)^2}{2\,\sigma_{e_m}^2\|\boldsymbol{C}\|^2}\right)dy \quad (27)$$

where $\mu_m = Tr\{\boldsymbol{G}_m\boldsymbol{C}\} \geq 0$

Using the Gaussian error function $(\cdot)$, the non outage probability can be further expressed as:

$$P_k = \begin{cases} \frac{1}{2} + \frac{1}{2}\text{erf}\left(\frac{\mu_k - \gamma_k\sigma_k^2}{\sqrt{2}\,\sigma_{e_k}\|\boldsymbol{Z}_k\|}\right), if\ \gamma_k\sigma_k^2 \leq \mu_k \\ \frac{1}{2} - \frac{1}{2}\text{erf}\left(\frac{\gamma_k\sigma_k^2 - \mu_k}{\sqrt{2}\,\sigma_{e_k}\|\boldsymbol{Z}_k\|}\right), if\ \gamma_k\sigma_k^2 \geq \mu_k \end{cases} \quad (28)$$

$$P_m = \begin{cases} \frac{1}{2} - \frac{1}{2}\text{erf}\left(\frac{\mu_m - k_m}{\sqrt{2}\,\sigma_{e_m}\|\boldsymbol{C}\|}\right), if\ k_m \leq \mu_m \\ \frac{1}{2} + \frac{1}{2}\text{erf}\left(\frac{k_m - \mu_m}{\sqrt{2}\,\sigma_{e_m}\|\boldsymbol{C}\|}\right), if\ k_m \geq \mu_m \end{cases} \quad (29)$$

Note that for any reliable communication link, the non-outage probabilities P_k and P_m must be close to one(with the ideal cases P_k and P_m =1). As the argument of the error function is positive for equations (28) and (29); the upper equation and lower equation corresponds to $P_k \geq \frac{1}{2}$ and $P_m \geq \frac{1}{2}$ respectively; whereas the lower equation and upper equation in (28) and (29) corresponds to $P_k \leq \frac{1}{2}$ and $P_m \leq \frac{1}{2}$ respectively. Therefore only the upper equation of (28) and the lower equation of (29) have to be considered. Using these equations, the non-outage probability constraints in (11) can be expressed as:

$$\text{erf}\left(\frac{\mu_k - \gamma_k\sigma_k^2}{\sqrt{2}\,\sigma_{e_k}\|\boldsymbol{Z}_k\|}\right) \geq 2p_k - 1 \quad (30)$$

$$\text{erf}\left(\frac{k_m - \mu_m}{\sqrt{2}\,\sigma_{e_m}\|\boldsymbol{C}\|}\right) \geq 2p_m - 1 \quad (31)$$

After simple mathematical manipulations, we can modify (30) and (31) as:

$$Tr(\boldsymbol{R}_k\boldsymbol{Z}_k) - \gamma_k\,\sigma_k^2 \geq b_k\|\boldsymbol{Z}_k\| \quad (32)$$

$$Tr(\boldsymbol{G}_m\boldsymbol{C})+b_m\|\boldsymbol{C}\| \ \leq k_m \quad (33)$$

where $b_k \triangleq \sqrt{2}\,\sigma_{e_k} erf^{-1}(2p_k - 1)$ k=1,........,K

$$b_m \triangleq \sqrt{2}\,\sigma_{e_m} erf^{-1}(2p_m - 1) \quad \text{m=1,.......,M}$$

Note that for $p_k < \frac{1}{2}$,the right hand side of (32) is negative and hence this constraint is satisfied automatically as far as $\gamma_k\,\sigma_k^2 \leq Tr(\boldsymbol{R}_k\boldsymbol{Z}_k)$;

we hereby consider the case where $p_k \geq \frac{1}{2}$, which is practically much more important than the case where $p_k < \frac{1}{2}$. similarly also, we will consider cases for $p_m \geq \frac{1}{2}$ .Thus the optimization problem of (11) can be written as:

$$\min_{\boldsymbol{w}_k} \sum_{k=1}^{K} Tr(\boldsymbol{W}_k)$$

subject to

$$\|\boldsymbol{Z}_k\| \ \leq \frac{1}{b_k}(Tr(\boldsymbol{R}_k\boldsymbol{Z}_k) - \gamma_k\,\sigma_k^2)$$

$$Tr(\boldsymbol{G}_m\boldsymbol{C})+b_m\|\boldsymbol{C}\| \ \leq k_m$$

$$\boldsymbol{W}_k = \boldsymbol{W}_k^H \text{ , rank } (\boldsymbol{W}_k)\text{=1 for k=1,.....,K} \quad (34)$$

where $\boldsymbol{Z}_k$ is given by (16); and $\boldsymbol{C}$ is given by (18). The constraint on the rank of $\boldsymbol{W}_k$ makes the optimization problem of (34) non-convex. We will transform this problem into a more convex optimization problem by replacing the rank-one constraint by the SDP constraints. Then the relaxed optimization problem can be written as:

$$\min_{\boldsymbol{w}_k} \sum_{k=1}^{K} Tr(\boldsymbol{W}_k) \text{ subject to}$$

$$\|\boldsymbol{Z}_k\| \leq t_{k;}\ \ t_k = \frac{1}{b_k}(Tr(\boldsymbol{R}_k\boldsymbol{Z}_k) - \gamma_k\,\sigma_k^2)$$

$$Tr(\boldsymbol{G}_m\boldsymbol{C})+b_m\|\boldsymbol{C}\| \ \leq k_m$$

$$\boldsymbol{W}_k \geq 0$$

$$\boldsymbol{W}_k = \boldsymbol{W}_k^H \text{ for all k=1,.....,K} \quad (35)$$

The optimization problem is now convex, and can be solved using some efficient numerical algorithm like seDUMI.

IV. SIMULATION RESULTS

Our simulation examples illustrate the performance of the proposed robust beamforming algorithm using similar approach to that of [17]. The channel covariance matrices for the PU and the SUs can be calculated using:

$$[R_k(\theta,\sigma_\theta)]_{pq} = e^{j\pi(p-q)sin\theta} e^{-(\pi(p-q)\sigma_\theta cos\theta)^2/2} \quad (36)$$

where θ is the central angle of the incoming rays to the *k*th users and *M*th users respectively, and the indices p and q represent the elements of the NXN covariance matrix. we assume that two secondary users are served by single base station (BS) equipped with uniform linear antenna array with eight antenna elements, there is also one active primary user. it is assumed that the primary user is located in 10° angular position relative to the secondary BS antenna broadside. While the first secondary user is considered to be at an angular position of 20°; the second secondary user is allowed to move between 25° and $50^\circ$of angular positions, all relative to the secondary BS antenna broadside. An angle spread of 2° around the main angular position is assumed for all users. We also assume a fixed noise variance of 1 at all users. we take SINR threshold $\gamma_k = 5dB$ and the IP threshold, $k_m = 4dB$. $p_k = p_m = p$ and $\sigma_{e_k}^2$=$\sigma_{e_m}^2 = \sigma_{e.}^2$ For each user central angle, the non-outage probability is calculated using the results of 1000 simulation runs with randomly changing error matrices. In the first part of the simulations, the proposed method (35) is tested for different values of $\sigma_{e.}^2$. In figure 2, $\sigma_\theta = 2^\circ$, and p=0.8 have been used; it can be seen that the secondary base station transmit power decreases as δ increases in the spatial separation between users. Figure 3 shows the total transmission power versus δ for the proposed and worst-case design methods with different values of $\sigma_{e.}^2$ We observed that the proposed method outperforms the worst-case design-based technique when the angle of separation is small. Figure 4 shows the plot of the transmitted power against the SINR; the plot shows that as the SINR increases, the transmitted power also increases both for the worst-case approach and the proposed approach respectively, however the proposed approach is more power efficient than the worst-case approach.

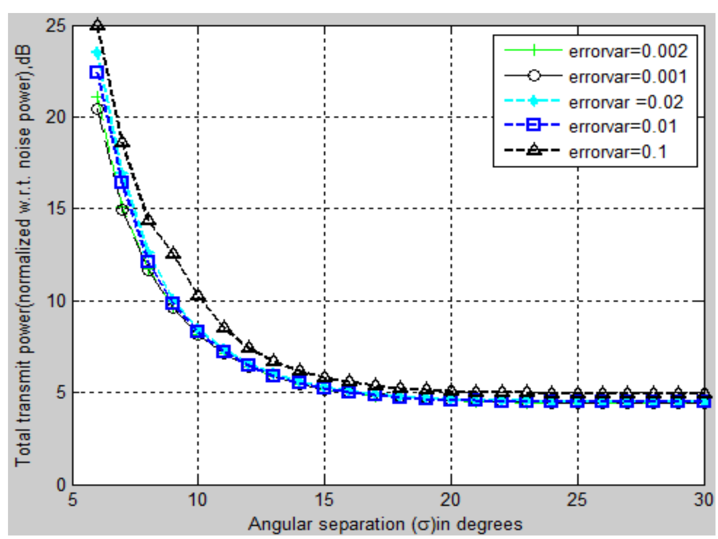

Fig. 2: Secondary BS transmit power of the robust method (35) versus angular separation for different values of $\sigma_{e.}^2$

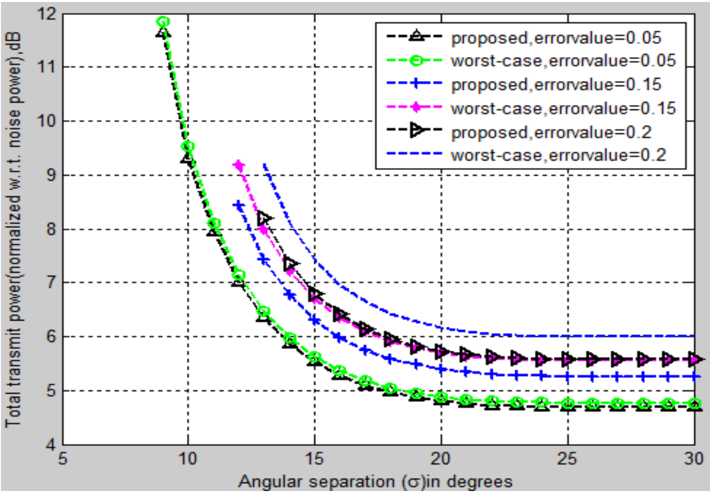

Fig. 3: Secondary BS transmit power of the robust methods (10)and (35) versus angular separation

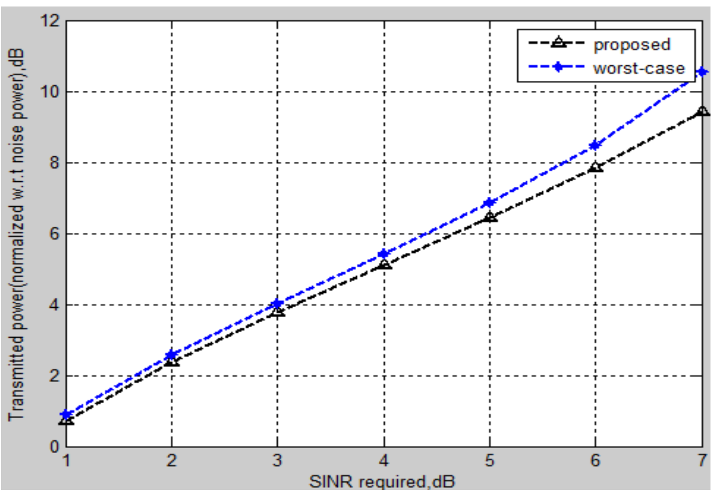

Fig. 4: Secondary BS transmit power of the robust methods (10) and (35) versus the SINR.

V. CONCLUSION

The proposed robust downlink beamforming methods have been seen to be more power efficient than the worst-case approach as shown by the simulation results. The technique minimizes the total transmit power while maintaining the non-outage probability for all users above a preselected threshold value. The proposed approach optimization problem was relaxed to a convex SDP problem a solved using the efficient interior point methods.

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