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Gas Turbine LQR, INTEGRAL Controllers and Optimal PID Tuning by Ant Colony Optimization Comparative Study

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Abstract– **Gas turbine is considered one of the most significant sources for power generation in the last decades, its dynamics analysis become increasingly more important. There are two types of controllers for gas turbines which are classical and intelligent controllers. The Linear Quadratic Controller (LQR) and integral control are classical type. Ant Colony Optimization (ACO) is intelligent type. The Linear Quadratic Controller (LQR), Integral control and Ant Colony Optimization (ACO) are developed for controlling the responses of a Gas Turbine system. Several researches of controllers and algorithms are presented to obtain the optimal solutions of the cost function and controller parameters. In this paper, an implementation of the Linear Quadratic Controller (LQR), integral control and Ant Colony Optimization (ACO) are performed for the optimization of cost function and controller parameters using an objective function. Proposed work focus on classical verses intelligent controllers. Simulation results for the systems are presented in time domain. The simulation is performed using the Linear Quadratic Controller (LQR), integral control and Ant Colony Optimization (ACO). The results of LQR are compared with the others types. The results show that the LQR is the best according to rise time and settling time.**

Index Terms– **Gas Turbine, Transfer function, LQR, Integral Control, PID, ITSE and Ant Colony Optimization (ACO)**

I. INTRODUCTION

IN electricity generation, an electric generator is a device that converts mechanical energy to electrical energy. A generator forces electric charge to flow through an external electrical circuit. The source of mechanical energy may be a reciprocating or turbine steam engine, water falling through a turbine or waterwheel, an internal combustion engine, a wind turbine, a hand crank, compressed air or any other source of mechanical energy. The dynamics analysis of gas turbines becomes increasingly more important. Shaft gas turbines have been utilized as prime movers for generators in utility and industrial power generation services for several years [1], [2], [3]. The PID controllers are frequently used in the control process of many different types of dynamic plants to regulate the time domain behavior. The PID controllers can usually provide a good closed loop response characteristic. There are several methods for tuning the PID parameters using several optimization algorithms. This paper focuses on optimal tuning of PID controller for the gas turbine machine using ACO algorithm and classical controller (LQR and Integral controller). ACO algorithm is one of the modern heuristic algorithms. Many performance estimation schemes are performed by ACO algorithm to obtain the optimum PID controller parameters of gas turbine system. Also LQR and integral are estimated in term of performance to obtain the optimal PID controller parameters. The paper is organized as follows. After this introduction, in Section II construction and designing of LQR are described in brief. In Section III construction and designing of integral control are described in brief. Section IV summarizes Ant Colony Optimization (ACO). In Section V, as a study case, the gas turbine generators are estimated and the step response of the derived model is simulated and discussed. Finally, section VI includes conclusion.

II. CONTROLLER DESIGN BY LQR

The objective in LQR design is to select the (SVFB) K that minimizes the performance index (J). The design procedure for finding the LQR feedback K is shown in Fig. 1 [7]. The designing steps as follows:

- Select design parameter matrices Q and R.
- Solve the algebraic Riccati equation for P.
- Find the SVFB using $K = R^{-1}B^T P$.

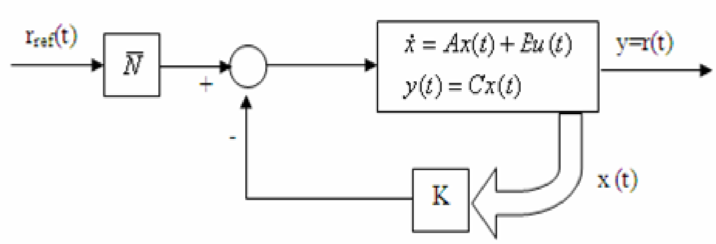

Fig. 1. the LQR Controller [7]

The system is described by Eq (1):

$$\dot{x} = Ax + Bu \qquad y = Cx \tag{1}$$

The performance index is defined by Eq (2):

$$J = \int_0^{\infty} (x^T Q x + u^T R u)\, dt \tag{2}$$

Where:

Q: $n \times n$ matrix.

R: $m \times m$ matrix.

R: a positive definite matrix.

Q: a positive semi definite matrix.

III. INTEGRAL CONTROL

The LQR control renders the origin of the system asymptotically stable [8], However, external disturbances and nonlinear effects present in the system may lead to poor closed loop system performance for the controls set point controllers next recall from the classical control theory that the integral control action yields zero steady state error for constant command input tracking even in the presence of exogenous disturbances thus, in this study, an integrator is added to the controller in the following way. Let *r* be the reference signal, the state (x) is considered as feedback for system, as well as the integral of the error in output by augmenting the plant state (x) with the extra 'integral state' as shown in Fig. 2 [7], and it is defined by the equation (3):

$$x_{n+1}(t) = \int_0^t (y(t) - r_d)\, dt \tag{3}$$

Since x_{n+1} satisfies the differential equation (4):

$$\dot{x}_{n+1} = y - r_d = Cx - r_d \tag{4}$$

Then:

$$\begin{bmatrix} \dot{x}_e(t) \\ \dot{x}e_{n+1}(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} x_e(t) \\ xe_{n+1}(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_e(t) \tag{5}$$

The closed loop system becomes:

$$\dot{e} = \hat{A} e + \hat{B} u_e \quad \text{where} \quad \hat{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}, \ \hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} \tag{6}$$

$$u = -K_x - K_1 x_{n+1} = -[K \quad K_1] \begin{bmatrix} x \\ x_{n+1} \end{bmatrix}$$

Where:

u is the control signal.

[K K_1] is the state feedback control.

The system has the (n+1) components.

IV. ANT COLONY OPTIMIZATION (ACO)

ACO is a class of algorithms, whose first member, called Ant System, was initially proposed by Colorni, Dorigo and Maniezzo [4]. The original idea comes from observing the exploitation of food resources among ants, in which ants' individually limited cognitive abilities have collectively been able to find the shortest path between a food source and the nest. An ant (called "blitz") runs more or less at random around the colony. If it discovers a food source, it returns more or less directly to the nest, leaving in its path a trail of pheromone; these pheromones are attractive, nearby ants will be inclined to follow, more or less directly, the track. Returning to the colony, these ants will strengthen the route. If there are two routes to reach the same food source then, in a given amount of time, the shorter one will be travelled by more ants than the long route. The short route will be increasingly enhanced, and therefore become more attractive.

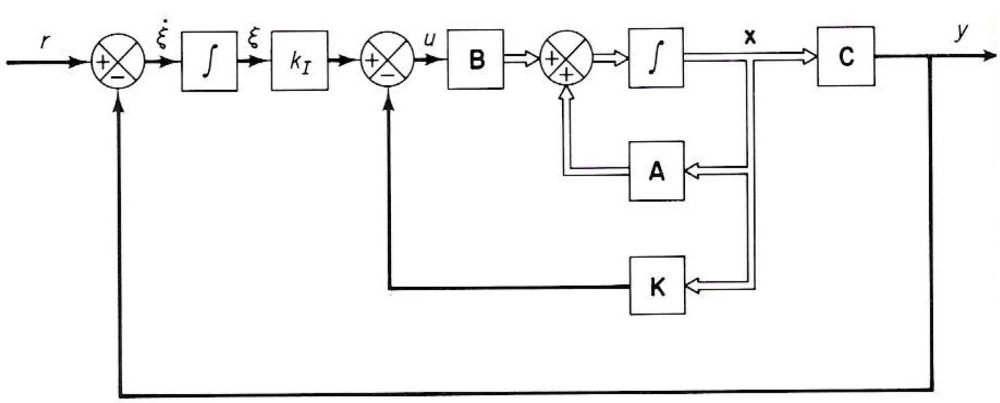

Fig. 2. Integral Control [7], [8]

The long route will eventually disappear because pheromones are volatile. Eventually, all the ants have determined and therefore "chosen" the shortest route [4], [5], [6].

ACO is depending upon the pheromone matrix $\tau = \{\tau_{ij}\}$ for the construction of good Solutions. The initial values of τ are:

$$\tau_{ij} = \tau_o \ \forall(i,j) \qquad where \ \tau o > 0 \tag{7}$$

The probability P_{ij}^{A}(t) of choosing a node *j* at a node *i* is defined in the equation (8). At each generation of the algorithm, the ant constructs a complete solution using this equation, starting at source node.

$$P_{ij}^{A}(t) = \frac{[\tau_{ij}(t)]^{\alpha}}{\sum_{i,j \in T^A}[\tau_{ij}(t)]^{\alpha}} \frac{[\eta_{ij}]^{\beta}}{[\eta_{ij}]^{\beta}} \quad \text{ifi, j} \in T^A \tag{8}$$

α and β = constants that determine the relative influence of the pheromone values and the heuristic values on the decision of the ant.

T^A= the path effectuated by the ant A at a given time.

The pheromone evaporation is a way to avoid unlimited increase of pheromone trails. Also it allows the forgetfulness of the bad choices:

$$\tau_{ij} = \rho\tau_{ij}(t-1) + \sum_{A=1}^{NA} \Delta\tau_{ij}^{A}(t) \tag{9}$$

Where $\Delta\tau_{ij}^{A}$ the quantity of pheromone on each path, NA: number of ants, ρ: the evaporation rate $0 < \rho \leq 1$. The quantity of pheromone $\Delta\tau_{ij}^{A}$ on each path may be defined as follows:

$$\Delta\tau_{ij}^{A} = \begin{cases} \frac{L^{min}}{L^A} & if \ i,j \ \epsilon T^A \\ 0 & \end{cases} \tag{10}$$

where, L^A = the value of the objective function found by the ant A.

L^{min} = the best solution carried out by the set of the ants until the current iteration.

The pheromone evaporation is a way to avoid unlimited increase of pheromone trails and also it allows the forgetfulness of the bad choices.

A. Implementation Algorithm

Step I: Initialize randomly potential solutions of the parameters Kp, Ti and Td by using uniform distribution. Initialize the pheromone trail and the heuristic value.

Step II: Place the A[th] ant on the node. Compute the heuristic value associated on the objective (minimize the error).

Step III: Use pheromone evaporation given by equation (9) to avoid unlimited increase of pheromone trails and allow the forgetfulness of bad choices.

Step IV: Evaluate the obtained solutions according to the objectives.

Step V: Display the optimum values of the optimization parameters.

Step VI: Update the pheromone, according to the optimum solutions calculated at step V: Iterate from step II until the maximum of iterations is reached

B. Tuning PID Controller Using Objective Ant Colony Algorithm

The objective function considered is based on the error criterion. The performance of a controller is best evaluated in terms of error criterion. A number of such criteria are available. Now, problem should be written as an optimization problem and then be solved. Selecting objective function is the most important part of this optimization problem. Because, choosing different objective functions may completely change the ant's variation state.

In optimization problem here, we use error signal. Between the reference input r (t) and the controlled plant output y (t) as the following equation: e (t) = y (t) – r(t).

V. SIMULATIONS

LV100 Gas Turbine Engine: Gas turbine engine (LV100) has two inputs (main fuel flow (W_f), turbine nozzle area (V_{atn}) and two outputs (gas generator speed (N), exhaust temperature (T) and consists of five states, the model of LV100 given by [1], [2]. The system is given by Eq (1):

u= [W_f V_{atn}]' (Input)

y= [N T]' (Output)

Where x is an *n*-vector (i.e. 5 x 1 matrix) containing the state variables, u is an *r*-vector of inputs and y is a *p*-vector of outputs. A, B, C, and D are respectively (5x5), (5x2), (2x5), and (2x2) matrices as shown in [1].

This paper studies the first output (N) according (u1), and the second output (T) according (u1).

a) LQR Control Design:

The system $\dot{x} = Ax + Bu$ with initial conditions x (0) =x_0

The performance index (PI) $J = \int_0^\infty (x^T Q x + u^T R u)\,dt$

The Algebra Riccati Equation (ARE) is defined by Eq (11):

$$A^T P + P A - P B R^{-1} B^T P + Q = 0 \tag{11}$$

Where (P) is the solution of the (ARE). The corresponding optimal gain is given as $K = R^{-1}B^T P$. And the minimum value of the performance index is $J = x^T(0)Px(0)$.In designing LQR controller, for speed gas turbine, LQR function in Matlab can be used to determine the value of the vector *K* which determined the feedback control law. This is done by choosing two parameter values, input *R = 1* and *Q = C^T*C*. For exhaust temperature of gas turbine choose Q =diag

([1 1 2000 1 2000]). In order to reduce steady state error of the system output, a value of constant gain N_r should be added after the reference, N_r given by Eq (12):

$$N_r = \frac{-1}{C.(A-B.K)^{-1}.B.} \qquad (12)$$

b) In designing integral controller based LQR choose the next values

Q=diag ([1700 1 2000 1 1 2000]); R=0.1; for speed and exhaust temperature of gas turbine system.

c) For tuning the parameters of PID controller by (ACO), the controller's Performance is take integral of time multiplied square error (ITSE)

$$F = \int_0^t te^2(t)\,dt \qquad (13)$$

ACO algorithm has the following parameters:

NA=20 (number of ants), Max Iteration=100.

α=1 (the relative influence of the pheromone value).

ρ=0.9 (the evaporation rate $0 < \rho \le 1$).

β=2(the heuristic values on the decision of the ant).

These values of ρ, α, β are selected after several testes see Table 1 and Table 2.

Initially, each parameter (Kp, Ti, Td) is randomly and uniformly distributed with an average value which is equal to the value founded by Ziegler-Nichols of rotor speed and exhaust temperature of the gas turbine model. After several iterations, the multi objective ant colony algorithm generates the best solutions of the PID parameters (KP, Ti, Td) of the rotor speed and the exhaust temperature for gas turbine system. The search space of these parameters is shown in Fig. 3 and Fig. 4 respectively.

Table 1: Rotor speed of gas turbine

α=1, β=2, NA=20		ρ=1, β=2, NA=20		α=1, ρ=1, NA=20	
ρ	Cost function ITSE	α	Cost function ITSE	β	Cost function ITSE
1	0.1250	1.2	0.2921	0.5	0.1489
0.9	0.1232	1.1	0.1933	1	0.1472
0.8	0.1244	1	0.1240	1.4	0.1359
0.7	0.1369	0.7	0.1550	1.8	0.1354
0.5	0.1392	0.5	0.1744	2	0.1352
0.3	0.1565	0.3	0.1941	2.2	0.1815
0.1	0.1601	0.1	0.2363	2.4	0.2550

Table 2: Exhaust temperature of gas turbine

α=1, β=2, NA=20		ρ=1, β=2, NA=20		α=1, ρ=1, NA=20	
ρ	Cost function ITSE	α	Cost function ITSE	β	Cost function ITSE
1	260.3916	1.2	253.0098	0.5	241.9932
0.9	249.1341	1.1	249.5297	1	237.9021
0.8	259.0675	1	247.3835	1.4	233.6754
0.7	266.0218	0.7	247.9062	1.8	202.1045
0.5	266.6057	0.5	248.2341	2	179.0678
0.3	266.6172	0.3	248.5824	2.2	212.1045
0.1	266.6391	0.1	251.7103	2.4	255.1643

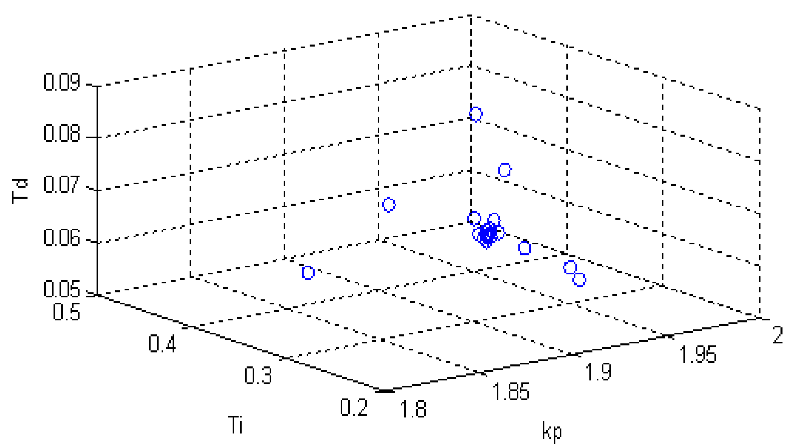

Fig. 3. Best values for PID Parameters (KP, Ti, Td) of the rotor speed

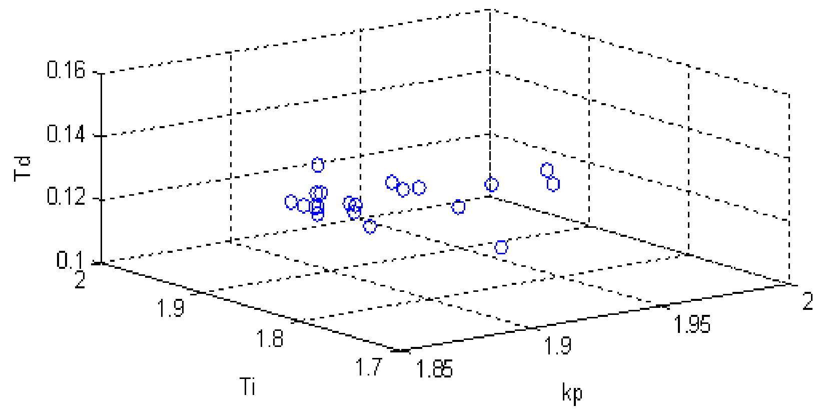

Fig. 4. Best values for PID parameters (KP, Ti, Td) of the exhaust temperature

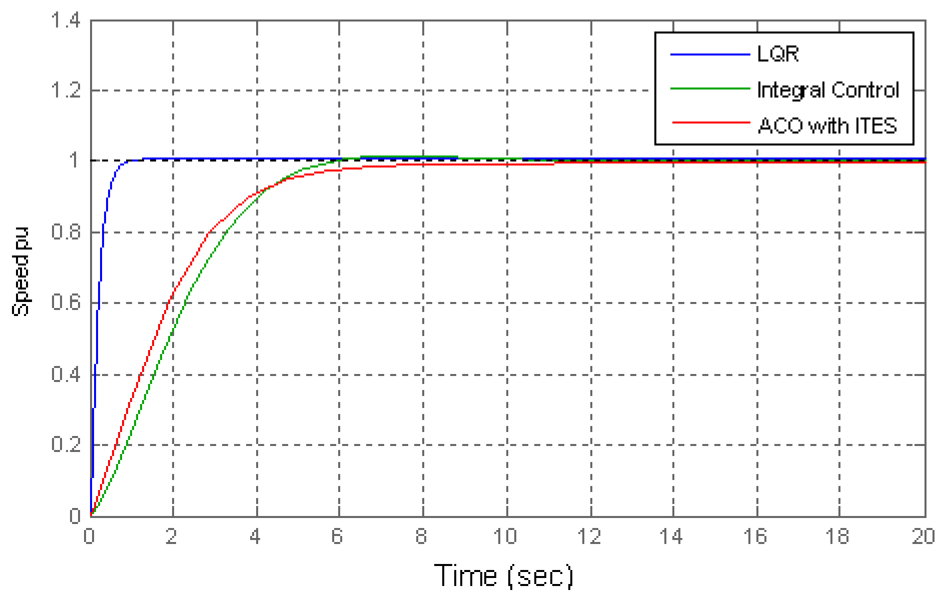

Fig. 5. Rotor speed of gas turbine system

Fig. 5 shows the simulations of the rotor speed for gas turbine system. The parameters of gas turbine model are given by [1], [2]. Table 3 includes response characteristic and time domain analysis obtained for rotor speed of LV100 gas turbine model using LQR, integral controllers and PID based ACO algorithm. Rise time, settling time, SSE, and overshoot are noted in the Table 3.

Table 3: Response characteristic of rotor speed for LV100 gas turbine

	Controller		
Response characteristic	**LQR**	**INTEGRAL**	**PID by ACO**
Rising Time (Tr)	0.390	3.58	3.45
Settling Time (Ts)	0.686	5.21	6.03
Percent Overshoot (OV)	0.002	0.0136	0
Steady-state Error (SSE)	0.005	0	0.0006

Fig. 6 shows the simulations of the exhaust temperature for gas turbine system, using the LQR, integral controllers and the ACO algorithm based ITSE cost function.

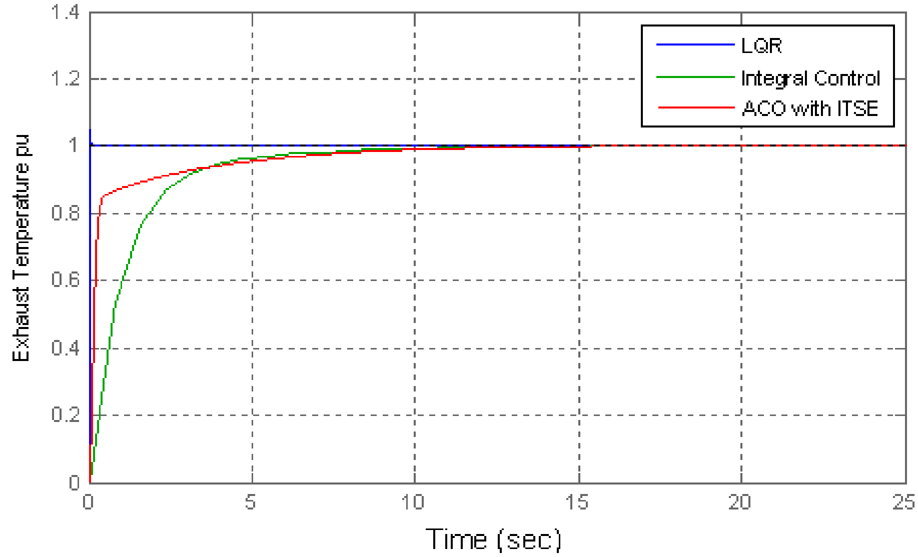

Fig. 6. Exhaust temperature of gas turbine system

Table 4 includes response characteristic and time domain analysis obtained for exhaust temperature of LV100 gas turbine model using LQR, integral controllers and PID based ACO algorithm. Rise time, Settling time, SSE, and overshoot are noted in the Table 4.

Table 4: response characteristic of exhaust temperature for LV100 gas turbine

	Controller		
Response characteristic	**LQR**	**INTEGRAL**	**PID byACO**
Rising Time (Tr)	0.0187	2.67	1.82
Settling Time (Ts)	0.0529	6.8	8.05
Percent Overshoot (OV)	0.0135	0.0230	0
Steady-state Error (SSE)	0.005	0	0.0005

Table 3 and Table 4 show the results for all the tested cases:

- The rise and settling time values are the minimum for LQR.
- The overshoot value is very small for LQR.
- The SSE value is very small for LQR.
- The SSE value is zero for integral as it is expected.
- The overshoot value is zero for PID tuning by ACO.
- The settling time value is the maximum for PID tuning by ACO.
- The rise time value is the maximum for integral control.

VI. CONCLUSION

The paper presents a several controllers in optimizing controller parameters to control the exhaust temperature, rotor speed of a gas turbine system and tuning the PID controller by ACO with ITSE performance criteria. Results show that the LQR is able to produce system responses with small rise time, small settling time, very small value of SSE and very small of overshoot. The integral control is able to produce system responses with high rise time, high settling time, very small of overshoot, and zero value of SSE. The proposed ACO-PID controller with ITSE performance criteria are able to produce system responses with high rise time, high settling time ,very small value of SSE and zero value of overshoot. Simulation and analysis results show that LQR controller relatively gives the better performance compared to Integral controller and ACO algorithm in controlling the exhaust temperature and rotor speed of a gas turbine system.

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