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Levelling with GPS

Dr. Nagi Zomrawi Mohammed
Associate Professor, College of Engineering, Sudan University of Science and Technology, Sudan
nagizomrawi@sustech.edu

***Abstract*– Levelling in surveying is the term stand for height difference determination between ground points. If a datum surface is defined for the observed points, the heights of these points (reduced levels) can be computed. In ordinary orthometric levelling Geoid surface is usually used as a datum. Levelling process can be carried out using different survey equipments and techniques. It may be precise; when precise optical or digital levels are used, or it may be ordinary; when using ordinary optical, automatic or digital levels. Moreover it can be carried out trigonometrically or barometrically. Now a day Global Positioning System (GPS) provides a quick modern technology to determine heights of points. But here, the datum surface is already defined with ellipsoidal surface. Therefore, different reduced levels can be expected compared with orthometric leveling. In this research work, difference in heights between numbers of points, in Khartoum State (Sudan), were carried out using both ordinary orthometric levelling (in which geoid is the reference datum) and GPS techniques (in which ellipsoid is the reference datum). Results obtained showed that the accuracy in levelling for GPS data in levelling for height differences is better than the accuracy of orthometric height determination. Therefore, GPS can be used in many engineering applications when variation in height is the main target. This result leads to ability of GPS to produce grid vector map at scale 1:5,000 and smaller. Moreover, GPS data can be reduced to a new arbitrary datum so that contour maps at scale 1:5,000 and smaller can be plotted.**

***Index Terms*– Leveling, GPS, Geoid and Ellipsoid**

I. INTRODUCTION

NUMBER of quantities has to be measured in usual survey works. These quantities may be bearings, angles, or distances (horizontal, vertical or inclined). Reduction of theses values leads to determination of relative position of the points.

Levelling is the name given to the process of measuring the difference in elevation between two or more points. The heights of points relative to a chosen surface are known as reduced levels of these points. Reference surface is usually known as a datum.

In engineering surveying, levelling has many applications and is used in all stages in construction projects from the initial site survey through to the final setting out. Contour maps, profiles and cross sections can be produced in all the stages of the project [2].

II. ORDINARY AND GEODETIC LEVELLING

In short distances, difference in level between two points can easily be determined by setting a staff vertically in each point. Then, constructing a horizontal line using optical or digital level. Difference in the intersection points of the horizontal line with both staves produce the difference in level between these ground points Fig. 1. This technique is simply known as ordinary leveling [4].

In long distances, when curvature of the earth has to be taken into account, levelling problem will be more complicated. This is so because horizontal line does not solve the problem directly. Levelling technique taking the curvature of the earth into account is usually known as geodetic levelling.

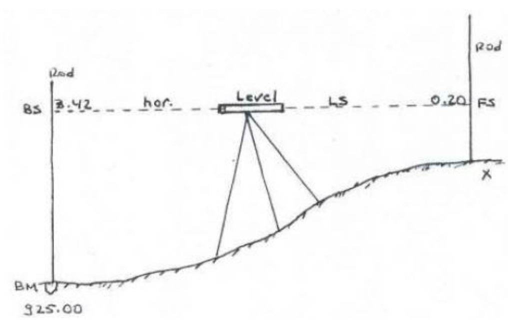

Fig. 1: Ordinary Leveling

III. GEOID AND ELLIPSOID

The *geoid* is an equipotential gravitational surface located approximately at mean sea level, which is everywhere perpendicular to the direction of gravity. *Orthometric* levelling is the term used when using geoid as a reference datum. Because of variations in the earth mass distribution, the geoid has an irregular shape [3].

Regular shape can be approximated by the *ellipsoid*. Ellipsoid is a mathematical surface obtained by revolving an ellipse about the earth's polar axis. Levelling based on taking the geoid as a reference datum is then termed g*eodetic* levelling. The dimensions of the ellipse are selected to give a good fit of the ellipsoid to the geoid over a large area, and are based upon surveys made in the area.

A two-dimensional view which illustrates conceptually the geoid and ellipsoid is shown in Fig. 2. As illustrated, the geoid contains non uniform undulations, and is therefore not readily defined mathematically. Ellipsoids, which approximate the geoid and can be defined mathematically, are therefore used to compute positions of widely spaced points that are located.

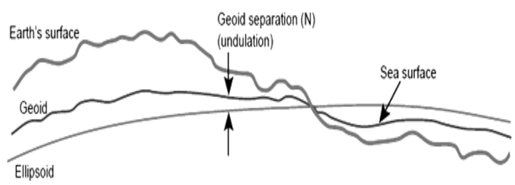

Fig. 2: Geoid and ellipsoid

Historically, *Clarke1880* ellipsoid was used as a reference datum -local datum- for long time in Sudan (Termed technically *Adindan Sudan*). Recently, because of the wide spread use of global positioning system GPS), world geodetic system (*WGS1984*) becomes an available substitute.

IV. GLOBAL POSITIONING SYSTEM

Global Positioning System (GPS) was developed by the (USA) to determine coordinates of points. Global positioning system consists of three components; these are space segment, control segment and user segment. Every component has its essential role in improving the positioning accuracy.

In this system, ground control receivers monitor the signals transmitted from a set of space segment (satellites). The received signals are used to solve for the coordinates of the position where the receivers are located. The GPS satellites are configured to provide the users with capability of positioning fixing.

Ground receiver equipments consist of two major units, antenna and units of analysis. The antenna is designed to receive the waves that come from satellites. Unit of analysis is linked to Antenna to analyze the data receiving by Antenna.

Global Positioning System (GPS) provides number of advantages e.g. the comprehensive coverage during the 24-hour, comprehensive coverage of spatial locations of each hemisphere, does not need to monitor the use of direct and traditional methods, link all the points of the coordinates of a global uniform and provide a great deal of time in the work of the main connecting points of the major projects.

Because of it is high precision measurements, it is uses not confined to the process of identifying the exact locations of the vessels at sea and ground control point locations, but many geodetic applications and uses of the system became an active role to play effective applications in the areas of civil engineering such as civil engineering, surveying, aerial surveys, environmental engineering , engineering of airports, air navigation, geodetic, geophysical applications and modern systems such as geographic information systems (GIS) etc.

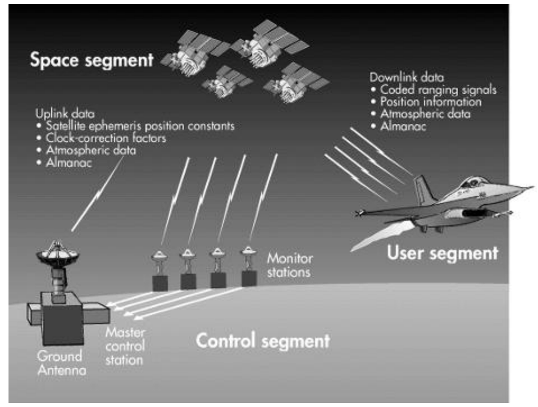

Fig. 3: GPS is composed of satellites, ground stations, and receivers

V. ACCURACY ESTIMATION

Statistically, average is the simplest quantity to represent a sample observation. Because it always affected by algebraic sign, the Root Mean Square Error (RMSE) become a suitable value to represent statistically a measurable quantity.

Accuracy estimation based on the criteria of the root mean squire error, can be computed as:

$$RMSE = \sqrt{\frac{\sum(\bar{x} - x)^2}{n}}$$

Where,

x is the actual quantity,
x is the measured quantity, and
n is the number of quantities.

VI. MEASUREMENTS AND RESULTS

In this research work, ordinary levelling was carried out for 49 points of about one kilometer interval along 50 kilometers in Khartoum state (Sudan). Automatic level was used to do the job. Level line was started at a known benchmark and end at another one. Level observations were reduced and adjusted. These level observations represent orthometric heights of the points (H) on Adindan Sudan reference datum as shown in Table 1.

Real Time GPS observations were carried out along the same line. Observations were processed and adjusted to compute ellipsoidal height (h) of all points reduced to WGS1984 reference datum. GPS levels of points were obtained as illustrated in Table 1.

Comparison between orthometric height of points and ellipsoidal heights of the same points can be carried out by computing the differences between both results as shown in the table. These differences represent the geoid ellipsoid separation (N) in the area. As well as it also represent the error in GPS levelling process.

Table 1: GPS heights, level heights, and errors in GPS heights observations

Point	GPS Heights (m)	Level Heights (m)	Errors in GPS (m)
1	386.662	382.935	3.727
2	387.588	383.797	3.791
3	387.772	384.117	3.655
4	388.761	385.13	3.631
5	389.445	385.862	3.583
6	388.974	385.472	3.502
7	390.653	386.967	3.686
8	390.406	386.707	3.699
9	389.78	386.172	3.608
10	389.43	386.157	3.273
11	389.469	386.202	3.267
12	389.721	386.438	3.283
13	390.092	386.785	3.307
14	389.263	385.945	3.318
15	389.46	386.12	3.34
16	390.072	386.713	3.359
17	390.318	386.958	3.36
18	391.613	388.225	3.388
19	393.554	390.108	3.446
20	396.098	392.587	3.511
21	398.293	394.757	3.536
22	399.01	395.472	3.538
23	399.472	395.947	3.525
24	398.838	395.324	3.514
25	399.095	395.554	3.541
26	400.235	396.719	3.516
27	399.903	396.347	3.556
28	400.268	396.666	3.602
29	398.081	394.464	3.617
30	395.394	391.711	3.683
31	393.896	390.263	3.633
32	395.277	391.571	3.706
33	396.884	393.224	3.66
34	398.659	394.964	3.695
35	399.075	395.52	3.555
36	398.962	395.302	3.66
37	401.600	397.928	3.672
38	405.936	402.263	3.673
39	407.63	403.978	3.652
40	410.558	406.843	3.715
41	410.698	406.98	3.718
42	407.92	404.245	3.675
43	404.546	401.823	2.723
44	401.535	397.809	3.726
45	401.214	397.449	3.765
46	404.248	400.449	3.799
47	407.558	403.749	3.809
48	406.925	403.191	3.734
49	405.875	402.161	3.714

Now, taking the GPS observations only, and computing the differences in levels between the successive points along the level line. GPS differences in heights were found to be as illustrated in Table 2.

Table 2: GPS differences in heights

Line	GPS differences in heights (m)	Line	GPS differences in heights (m)
1-2	-0.926	25-26	-1.14
2-3	-0.184	26-27	0.332
3-4	-0.989	27-28	-0.365
4-5	-0.684	28-29	2.187
5-6	0.471	29-30	2.687
6-7	-1.679	30-31	1.498
7-8	0.247	31-32	-1.381
8-9	0.626	32-33	-1.607
9-10	0.35	33-34	-1.775
10-11	-0.039	34-35	-0.416
11-12	-0.252	35-36	0.113
12-13	-0.371	36-37	-2.638
13-14	0.829	37-38	-4.336
14-15	-0.197	38-39	-1.694
15-16	-0.612	39-40	-2.928
16-17	-0.246	40-41	-0.14
17-18	-1.295	41-42	2.778
18-19	-1.941	42-43	3.374
19-20	-2.544	43-44	3.011
20-21	-2.195	44-45	0.321
21-22	-0.717	45-46	-3.034
22-23	-0.462	46-47	-3.31
23-24	0.634	47-48	0.633
24-25	-0.257	48-49	1.05

Differences in heights between the same successive points are also computed for ordinary levelling observations. Results are tabulated as shown in Table 3.

Table 3: Level differences in heights

Line	Level differences in heights (m)	Line	Level differences in heights (m)
1-2	-0.862	25-26	-1.165
2-3	-0.32	26-27	0.372
3-4	-1.013	27-28	-0.319
4-5	-0.732	28-29	2.202
5-6	0.39	29-30	2.753
6-7	-1.495	30-31	1.448
7-8	0.26	31-32	-1.308
8-9	0.535	32-33	-1.653
9-10	0.015	33-34	-1.74
10-11	-0.045	34-35	-0.556
11-12	-0.236	35-36	0.218
12-13	-0.347	36-37	-2.626
13-14	0.84	37-38	-4.335
14-15	-0.175	38-39	-1.715
15-16	-0.593	39-40	-2.865
16-17	-0.245	40-41	-0.137
17-18	-1.267	41-42	2.735
18-19	-1.883	42-43	2.422
19-20	-2.479	43-44	4.014
20-21	-2.17	44-45	0.360
21-22	-0.715	45-46	-3.000
22-23	-0.475	46-47	-3.300
23-24	0.623	47-48	0.558
24-25	-0.230	48-49	1.030

Now, Errors in differences in heights obtained from GPS levelling and that obtained from ordinary levelling can be can be computed by subtracting the differences in heights obtained by each method. Table 4 represents errors in GPS height differences.

Table 4: Errors in GPS height differences

Line	GPS Height Differences	Level Height Differences	Errors in GPS Height Differences
1-2	-0.926	-0.862	-0.064
2-3	-0.184	-0.32	0.136
3-4	-0.989	-1.013	0.024
4-5	-0.684	-0.732	0.048
5-6	0.471	0.39	0.081
6-7	-1.679	-1.495	-0.184
7-8	0.247	0.26	-0.013
8-9	0.626	0.535	0.091
9-10	0.35	0.015	0.335
10-11	-0.039	-0.045	0.006
11-12	-0.252	-0.236	-0.016
12-13	-0.371	-0.347	-0.024
13-14	0.829	0.84	-0.011
14-15	-0.197	-0.175	-0.022
15-16	-0.612	-0.593	-0.019
16-17	-0.246	-0.245	-0.001
17-18	-1.295	-1.267	-0.028
18-19	-1.941	-1.883	-0.058
19-20	-2.544	-2.479	-0.065
20-21	-2.195	-2.17	-0.025
21-22	-0.717	-0.715	-0.002
22-23	-0.462	-0.475	0.013
23-24	0.634	0.623	0.011
24-25	-0.257	-0.23	-0.027
25-26	-1.14	-1.165	0.025
26-27	0.332	0.372	-0.04
27-28	-0.365	-0.319	-0.046
28-29	2.187	2.202	-0.015
29-30	2.687	2.753	-0.066
30-31	1.498	1.448	0.05
31-32	-1.381	-1.308	-0.073
32-33	-1.607	-1.653	0.046
33-34	-1.775	-1.74	-0.035
34-35	-0.416	-0.556	0.140
35-36	0.113	0.218	-0.105
36-37	-2.638	-2.626	-0.012
37-38	-4.336	-4.335	-0.001
38-39	-1.694	-1.715	0.021
39-40	-2.928	-2.865	-0.063
40-41	-0.14	-0.137	-0.003
41-42	2.778	2.735	0.043
42-43	3.374	2.422	0.952
43-44	3.011	4.014	-1.003
44-45	0.321	0.36	-0.039
45-46	-3.034	-3	-0.034
46-47	-3.31	-3.3	-0.01
47-48	0.633	0.558	0.075
48-49	1.05	1.03	0.02

Table 4 below represents the root mean square error in height differences. Note that the RMSE in GPS heights was found to be 3.83m, where, RMSE in GPS height differences was found to be 0.213m.

Table 4: RMSE of height differences

Differences	Level differences in heights (m)	GPS differences in heights (m)	Errors in GPS Height (m)	Errors in GPS Height Difference (m)
RMSE	**1.679**	**1.683**	**3.569**	**0.213**

Fig. 1 illustrates diagrammatically the RMSE of height difference in each case.

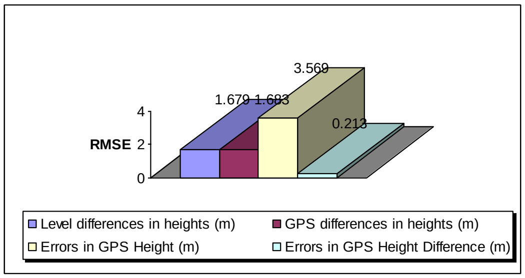

Fig. 1: RMSE of height differences

VII. CONCLUSION

Referring to the results obtained above and analysis carried out. It could be directly conclude that:

i). Success of direct utilization of GPS in levelling observations depends on the value of geoid ellipsoid separation in the area. This separation was estimates in the study area with 3.569m. This value can be expected as an error when using GPS directly in levelling applications.

ii). GPS produces better results in the case when differences in heights become the main objective. This is the case of cross-sections production, profiling and most of engineering applications.

iii). The Accuracy of GPS in height differences determination was found to be 0.213m, the value that allow production of grid vector maps at scale 1: 5,000 and smaller according to national specifications.

iv). Moreover, GPS data can be reduced to a new arbitrary datum so that contour maps at scale 1:5,000 and smaller can be plotted.

VIII. RECOMMENDATIONS

This study was oriented to evaluate the utilization of GPS in direct levelling applications. Further research works can be oriented to:

i). Try to improve the accuracy of GPS in height difference determination.

ii). Determine precisely the geoid model in the area.

iii). Evaluate the Integration of GPS and ordinary levels in one project.

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