> Least Square Curve Fitting Applications under Rest State Environment in Internet Traffic Sharing in Computer Network

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#### Abstract

While dealing with internet users in the setup of two operators the relationship between traffic sharing and blocking probability is examined by many authors using Markov chain model. Assuming dial-up-setup of connection network and introduction of rest state the mathematical relationship has been modified. This relationship contains many input parameters of the model with special reference to rest state parameter. This paper presents a much simplified form of relationship between blocking probability and traffic sharing under rest state. Because of variation of model parameter there is always an expected change in the equation of straight line. All these changes are average out to get best average linear relationship between traffic sharing and blocking probability. This one is useful for immediate computations of traffic sharing levels when input has variant values.


Index Terms- Transition Probability Matrix (TPM), Markov Chain Model (MCM), Coefficient of Determination (COD) and Confidence Interval

## I. INTRODUCTION

THE Internet is one of the most widely used tool for accessing the digital information. This service may be implemented through broadband or dial-up-setup. Many developing countries are still having dial-up-setup for market connections. Naldi (2002) has suggested a relationship between traffic sharing and blocking probability in a network using Markov chain model under the assumption of two networks operators. Shukla and Thakur (2010) extended the approach by introducing a rest state in the structural setup. The modified form of relationship contains many parameters therefore the actual simplified form is difficult to understand. In this paper an attempt has been made to express the actual relationship between traffic sharing and blocking probability in linear simplified form.

## II. A REVIEW

The stochastic process has been used by many scientists and researchers for the purpose of statistical modeling whose detailed description is in Medhi $(1991,1992)$. Chen and Mark (1993) discussed the fast packet switch shared concentration and output queueing for a busy channel. Humbali and Ramani
(2002) evaluated multicast switch with a variety of traffic patterns. Newby and Dagg (2002) have a useful contribution on the optical inspection and maintenance for stochastically deteriorating system. Dorea et al. (2004) used Markov chain for the modelling of a system and derived some useful approximations. Yeian and Lygeres (2005) presented a work on stabilization of class of stochastic different equations with Markovian switching. Shukla et al. (2007 a) advocated for model based study for space division switches in computer network. Shukla et al. $(2007$ b) presented crime based user analysis in internet traffic sharing under cyber crime. Francini and Chiussi (2002) discussed some interesting features for QoS guarantees to the unicast and multicast flow in multistage packet switch.

On the reliability analysis of network a useful contribution is by Agarwal and Lakhwinder (2008) whereas Paxson (2004) introduced some of their critical experiences while measuring the internet traffic. Shukla et al. (2009 a, b and c) presented different dimensions of internet traffic sharing in the light of share loss analysis and comparison of method for internet traffic sharing. Shukla et al.(2009) studied rest state analysis in internet traffic distribution in multi-operator environment. Shukla and Thakur (2009) discussed a modeling of behavior of cyber criminals when two internet operators in markets. Shukla et al. (2009) studied and discussed Markov chain model for the analysis of round robin scheduling and state probability analysis of internet traffic sharing.

Shukla et al. (2010 a, b. c, d, e and f) have given some Markov Chain model applications in view to disconnectivity factor, multi marketing and crime based analysis. Shukla et al. (2010) presented Index based internet traffic analysis of users by a Markov chain model. Shukla et al. (2010 a, b, c and d) discussed cyber crime analysis for multidimensional effect in computer network and internet traffic sharing. Shukla et al. (2010) presented ISO-Share analysis of internet traffic sharing in presence of favoured disconnectivity. Shukla et al. (2011 a, $\mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}, \mathrm{f}$ and g ) discussed the elasticity property and its impact on parameters of internet traffic sharing in presence blocking probability of computer network specially when two operators are in business competitions with each other in a market. Shukla, Tiwari and Thakur (2011) presented analysis of internet traffic distribution for user behavior based
probability in multi-market environment. Shukla et al. (2011) presented analysis of user web browsing using Markov chain model for ISO-browser share probability. Shukla et al. (2012) studied least square curve fitting for ISO-Failure in Web Browsing using Markov Chain Model. Shukla, Verma and Gangele (2012) presented Least Square Based Curve Fitting in Internet Access Traffic Sharing in Two Operator Environment.

## III. USERS BEHAVIOUR AS SYSTEM

Consider following hypotheses for the behaviour of user, with rest, blocking, and initial choice parameters, while sharing the traffic between the two operators.

- The competitive market has a café, containing Internet facility of operators $\mathrm{O}_{1}$ and $\mathrm{O}_{2}$.
- A user enters into café with initial choice (first choice) p and ( $1-\mathrm{p}$ ) for $\mathrm{O}_{1}$ and $\mathrm{O}_{2}$ respectively $(0 \leq \mathrm{p} \leq 1)$.
- The p is affected by advertising, marketing, quality-ofservice and past preference (or attractiveness).
- The premise of café has a place for human energy recharge (like, rest, entertainment, games, refreshment etc.) denoted as $R$, with probability $\mathrm{p}_{\mathrm{R}}$.
- After each failed call attempt, the user has three choices:
i. he can abandon with probability $\mathrm{p}_{\mathrm{A}}$,
ii. switch over to other operator for a new attempt or moves for a little rest (on R).
- From R, user switches to either of operators [with probability $r$ and (1-r)] but can not abandon.
- Switching among $\mathrm{O}_{1}, \mathrm{O}_{2}$ and R is on a call-by-call basis depending just on the latest attempt. The physical movement of user is also an attempt.
During the repeated call, the blocking probability offered by $\mathrm{O}_{1}$ is $\mathrm{L}_{1}$ and of $\mathrm{O}_{2}$ is $\mathrm{L}_{2}$. The blocking implies situation when call attempt process fails to connect an operator.


## IV. MARKOV CHAIN MODEL

These types models are used by Shukla et. al. (2007), Shukla and Gadewar (2007) in switch architecture analysis in computer network.

Under hypotheses of user's behaviour and user's attitude can be modelled by a five-state discrete-time Markov chain $\left\{X^{(n)}, n \geq 0\right\}$ such that $X^{(n)}$ stands for the state of random variable $X$ at $n^{\text {th }}$ attempt (call or movement) made by a user over state space $\left\{O_{1}, O_{2}, R, Z, A\right\}$, where
State $O_{1}$ : first operator
State $O_{2}$ : second operator
State $R$ : temporary rest for a short time
State $Z$ : success (in connectivity)
State $A$ : abandon the call connectivity attempt process.

The Fig. 1 explains the diagrammatic form of transition and Fig. 2 is transition probability matrix of order 6X6 of this model.


Fig. 1. (Transition model) [Using Shukla and Thakur (2010)]

## V. COMPUTATION OF TRANSITION PROBABILITIES BETWEEN STATES

(i) The initial probabilities (initial choice) for user to start with from any operators

$$
P\left[X^{(0)}=O_{1}\right]=p, P\left[X^{(0)}=O_{2}\right]=(1-p) \ldots(5.1)
$$

(ii) If in $(n-1)^{\text {th }}$ attempt, call for $O_{1}$ blocked, and user abandons the process.

$$
\begin{equation*}
P\left[X^{(n)}=A / X^{(n-1)}=O_{1}\right] \tag{5.2}
\end{equation*}
$$

(iii) P [blocked at $\left.\mathrm{O}_{1}\right] \mathrm{P}$ [ abandon the process ]

$$
=\mathrm{L}_{1} \mathrm{p}_{\mathrm{A}}
$$

Similar for $O_{2}$,

$$
\begin{equation*}
P\left[X^{(n)}=A / X^{(n-1)}=O_{2}\right]=\mathrm{L}_{2} \mathrm{p}_{\mathrm{A}} \tag{5.3}
\end{equation*}
$$

(iv) At $O_{1}$ in $n^{\text {th }}$ attempt, call is successful only when call does not block in $(n-1)^{\text {th }}$ and user is at Z in the next.
$P\left[X^{(n)}=Z / X^{(n-1)}=O_{1}\right]$
$=\mathrm{P}$ [not blocked at $\mathrm{O}_{1]}=\left(1-\mathrm{L}_{1}\right)$
Similar for $O_{2}$,
$P\left[X^{(n)}=Z / X^{(n-1)}=O_{2}\right]=\left(1-\mathrm{L}_{2}\right)$
(v) At $O_{1}$, when call blocked in $(n-1)^{\text {th }}$ attempt, user does not want to abandon, but wants a little rest then,

$$
\begin{equation*}
P\left[X^{(n)}=R / X^{(n-1)}=O_{1}\right] \tag{5.6}
\end{equation*}
$$

$=\mathrm{P}$ [blocked at $\left.\mathrm{O}_{1}\right] \mathrm{P}[$ not abandon $] \mathrm{P}[$ a little rest $]$
$=\mathrm{L}_{1}\left(1-\mathrm{p}_{\mathrm{A}}\right) \mathrm{p}_{\mathrm{R}}$
Similar to $O_{2}$,
$P\left[X^{(n)}=R / X^{(n-1)}=O_{2}\right]=\mathrm{L}_{2}\left(1-\mathrm{p}_{\mathrm{A}}\right) \mathrm{p}_{\mathrm{R}}$
At $O_{1}$, if call is blocked in $(n-1)^{\text {th }}$ attempt, user does not want both abandon and rest, then he shifts to $O_{2}$.
$P\left[X^{(n)}=O_{2} / X^{(n-1)}=O_{1}\right]$
$=\mathrm{P}$ [blocked at $\left.\mathrm{O}_{1}\right] \mathrm{P}$ [not abandon] $\mathrm{P}[$ not rest $]$ $=L_{1}\left(1-p_{\mathrm{A}}\right)\left(1-\mathrm{p}_{\mathrm{R}}\right)$

Similar to $O_{2}$,
$P\left[X^{(n)}=O_{1} / X^{(n-1)}=O_{2}\right]=\mathrm{L}_{2}\left(1-\mathrm{p}_{\mathrm{A}}\right)\left(1-\mathrm{p}_{\mathrm{R}}\right) \ldots$
(vi) Also, assume for, $0 \leq r \leq 1$

$$
\left.\begin{array}{l}
P\left[X^{(n)}=O_{1} / X^{(n-1)}=R\right]=r \\
P\left[X^{(n)}=O_{2} / X^{(n-1)}=R\right]=1-r \tag{5.10}
\end{array}\right\} \cdots
$$



Fig. 2. (Transition Probability Matrix) [using Shukla and Thakur 2010)]

## VI. SOME RESULTS FOR $n^{\text {th }}$

Theorem 6.1: If user restricts to only $O_{1}$ and R then $\mathrm{n}^{\text {th }}$ attempt state probabilities are:

$$
\begin{equation*}
P\left[X^{(2 n)}=O_{1}\right]=p E^{n}, P\left[X^{(2 n+1)}=O_{1}\right]=0 \tag{6.1}
\end{equation*}
$$

Where $E=B_{1} r, B_{1}=L_{1}\left(1-p_{A}\right) p_{R}$
Theorem 6.2: If user restricts to only $O_{2}$ and $R$ then $n^{\text {th }}$ attempt state probabilities are:
$P\left[X^{(2 n)}=O_{2}\right]=(1-p) D, P\left[X^{(2 n+1)}=O_{2}\right]=0 \ldots$
Where, $D=B_{2}(1-r), B_{2}=L_{2}\left(1-p_{A}\right) p_{R}$
Theorem 6.3: If user restricts to only between $O_{1}$ and $O_{2}$, not interested for $R$ then,
$\left.\begin{array}{l}P\left[X^{(2 n)}=O_{1}\right]=p C^{n} \\ P\left[X^{(2 n+1)}=O_{1}\right]=(1-p) A_{2} C^{n} \\ P\left[X^{(2 n)}=O_{2}\right]=(1-p) C^{n} \\ P\left[X^{(2 n+1)}=O_{2}\right]=p A_{1} C^{n}\end{array}\right\}$
Where $C=A_{1} A_{2}, A_{1}=L_{1}\left(1-p_{A}\right)\left(1-p_{R}\right)$,
$A_{2}=L_{2}\left(1-p_{A}\right)\left(1-p_{R}\right)$

Theorem 6.4: If call attempt is among $O_{1}, O_{2}$ and $R$ only then for $n^{\text {th }}$ state probability the approximate expressions of probabilities are,

$$
\left.\begin{array}{l}
P\left[X^{(2 n)}=O_{1}\right]=p(C+E)^{n} \\
P\left[X^{(2 n+1)}=O_{1}\right]=(1-p) A_{2}(C+D+E)^{n} \\
P\left[X^{(2 n)}=O_{2}\right]=(1-p)(C+D)^{n}  \tag{6.4}\\
P\left[X^{(2 n+1)}=O_{2}\right]=p A_{1}(C+D+E)^{n}
\end{array}\right\} .
$$

## VII. TRAFFIC SHARE OVER LARGE NUMBER OF ATTEMPTS

Suppose n is large, then $\bar{P}_{i}=\left[\lim _{n \rightarrow \infty} \bar{P}_{i}^{(n)}\right], \mathrm{i}=1,2$ and
$\left.\begin{array}{l}{\left[\bar{P}_{1}\right]_{F U}=\frac{\left(1-L_{1}\right) p}{1-E}} \\ {\left[\bar{P}_{2}\right]_{F V}=\frac{\left(1-L_{2}\right)(1-p)}{1-D}} \\ {\left[\bar{P}_{1}\right]_{P V}=\left(1-L_{1}\right)\left[\frac{p+(1-p) A_{2}}{1-C}\right]} \\ {\left[\bar{P}_{2}\right]_{P V}=\left(1-L_{2}\right)\left[\frac{(1-p)+p A_{1}}{1-C}\right]} \\ {\left[\bar{P}_{1}\right]_{C U}=\left(1-L_{1}\right)\left[\frac{p}{1-(C+E)}+\frac{(1-p) A_{2}}{1-(C+D+E)}\right]} \\ {\left[\bar{P}_{2}\right]_{C V}=\left(1-L_{2}\right)\left[\frac{1-p}{1-(C+D)}+\frac{p A_{1}}{1-(C+D+E)}\right]}\end{array}\right\}$

## VIII. LEAST SQUARE CURVE FITTING

We suggest a linear relationship where $\mathrm{a}, \mathrm{b}$ are constants $\bar{P}_{1}=\hat{a}+\hat{b} \cdot L_{1}$
Let $\left(\left(\bar{P}_{1 i}, L_{1 i}\right) i=1,2,3 \ldots . n\right.$ be n observations generated from $\left[\overline{P_{1}}\right]_{F U}$ and $\left[\overline{P_{1}}\right]_{P I U}$ of equation (7.1) keeping values fixed for $\mathrm{p}, \mathrm{p}_{\mathrm{A}}$ and $\mathrm{L}_{2}$. Suppose $\mathrm{n}=9$ and blocking probabilities for $\mathrm{L}_{1}$ are $(0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8$, 0.9 ) then using (8.1), the generated data of $\left[\overline{P_{1}}\right]_{F U}$ and $\left[\overline{P_{1}}\right]_{P I U}$ are is in table (9.1, 9.2, 9.3, 9.4, 9.5 and 9.6). The estimated $\hat{P}_{1}$ is obtained using line equation (9.1) with values of $\hat{a}, \hat{b}$.

## XI. FITTING THE STRAIGHT LINE

We suggest an approximate the relationship between parameter $\left[\overline{P_{1}}\right]_{F U}$ and $\left[\overline{P_{1}}\right]_{P I U}$ and $\mathrm{L}_{1}$ through a straight line $\left[\overline{P_{1}}\right]_{F U}=\mathrm{a}+\mathrm{b} . \mathrm{L}_{1}$ and $\left[\overline{P_{1}}\right]_{P I U}=\mathrm{a}+\mathrm{b} . \mathrm{L}_{1}$. The normal equations are

$$
\left.\begin{array}{l}
\sum_{i=1}^{n} P_{1 i}=n \cdot a+b \sum_{i=1}^{n} L_{1}  \tag{9.1}\\
\sum P_{1 i} \cdot L_{1 i}=a \sum_{i=1}^{n} L_{1}+b \sum_{i=1}^{n} L_{1}^{2}
\end{array}\right\}
$$

Where $\mathrm{P}_{\mathrm{li}}=\left[\overline{P_{1}}\right]_{F U}$ or $\left[\overline{P_{1}}\right]_{P I U}$

Solving the above equation the least square estimate are a and b are (denoted as $\hat{a}, \hat{b}$ ):

$$
\begin{align*}
& \hat{a}=\left\{\frac{1}{n} \sum_{i=1}^{n} P_{1 i}-\hat{b} \sum_{i=1}^{n} L_{1}\right\}  \tag{9.2}\\
& \hat{b}=\left\{\frac{n \sum_{i=1}^{n} P_{1 i} L_{1}-\left(\sum_{i=1}^{n} P_{1}\right)\left(\sum L_{1}\right)}{n \sum_{i=1}^{n} L_{1}^{2}-\left(\sum_{i=1}^{n} L_{1}\right)^{2}}\right\} \tag{9.3}
\end{align*}
$$

Where n is the number of observations in sample ( n ) and the coefficient of determination (COD) is defined as
$\mathrm{COD}=\left\{\frac{\sum\left(\hat{P}_{1 i}-\bar{P}_{1}\right)^{2}}{\sum\left(P_{1 i}-\bar{P}_{1}\right)^{2}}\right\}$
where $\bar{P}_{1}=\frac{1}{n} \sum P_{1 i}$ is mean of original data of $\mathrm{P}_{1}$ obtained through Markov chain model. The term $\hat{P}_{1}=\hat{a}+\hat{b} \cdot L_{1}$ is the estimated value given observation $\mathrm{L}_{1}$. The COD lies between 0 to 1 . If the straight line is good fit then it is near to 1 . We generate pair of value ( $\bar{P}_{1}, \mathrm{~L}_{1}$ ) from express tables (9.1, 9.2, $9.3,9.4,9.5$ and 9.6 ) by providing values of fixed input parameters.

Table 9.1, 9.2, 9.3 are based on $\left[\overline{P_{1}}\right]_{F U}$ and 9.4, 9.5, 9.6 are for $\left[\overline{P_{1}}\right]_{P I U}$ where
$\left[\bar{P}_{1}\right]_{F U}=\frac{\left(1-L_{1}\right) p}{1-E}$
$\left[\bar{P}_{1}\right]_{P I U}=\left(1-L_{1}\right)\left[\frac{p+(1-p) A_{2}}{1-C}\right]$

Table 9.1: [Values of $\mathrm{L}_{1},\left[\mathrm{P}_{1}\right]_{\mathrm{FU}}$ and $\left.\left[\hat{P}_{1}\right]_{\mathrm{FU}}\right]$ when $\mathrm{p}=0.4, \mathrm{~L}_{2}=0.3, \mathrm{p}_{\mathrm{A}}=0.2$

| Fixed <br> parameter | $\mathrm{L}_{1}$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | COD |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{p}=0.4$ <br> $\mathrm{~L}_{2}=0.3$ <br> $\mathrm{p}_{\mathrm{A}}=02$ | $\mathrm{P}_{1}$ | 0.3719 | 0.3418 | 0.3097 | 0.2752 | 0.2380 | 0.1980 | 0.1546 | 0.1075 | 0.0561 |  |
|  | $\hat{P}_{1}$ | 0.3850 | 0.3458 | 0.3065 | 0.2673 | 0.2281 | 0.1889 | 0.1496 | 0.1104 | 0.0712 | $\mathbf{0 . 9 9 2 5}$ |

$$
\begin{equation*}
\hat{a}=0.4242 ; \quad \hat{b}=-0.3922 ; \quad \hat{P}_{1}=0.4242-0.3922 .\left(L_{1}\right) \tag{9.6.1}
\end{equation*}
$$

Table 9.2: [Values of $\mathrm{L}_{1},\left[\mathrm{P}_{1}\right]_{\mathrm{FU}}$ and $\left.\left[\hat{P}_{1}\right]_{\mathrm{FU}}\right]$ when $\mathrm{p}=0.4, \mathrm{~L}_{2}=0.5, \mathrm{p}_{\mathrm{A}}=0.5$

| Fixed <br> parameter | $\mathrm{L}_{1}$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | COD |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{p}=0.4$ <br> $\mathrm{~L}_{2}=0.5$ <br> $\mathrm{p}_{\mathrm{A}}=0.5$ | $\mathrm{P}_{1}$ | 0.3673 | 0.3333 | 0.2978 | 0.2608 | 0.2222 | 0.1818 | 0.1395 | 0.0952 | 0.0487 |  |
|  | $\hat{P}_{1}$ | 0.3752 | 0.3355 | 0.2958 | 0.2560 | 0.2163 | 0.1765 | 0.1368 | 0.9712 | 0.0573 | $\mathbf{0 . 9 9 7 5}$ |

$$
\begin{equation*}
\hat{a}=0.4150 ; \quad \hat{b}=-0.3973 ; \quad \hat{P}_{1}=0.4150-0.3973 .\left(L_{1}\right) \tag{9.6.2}
\end{equation*}
$$

Table 9.3: [Values of $\mathrm{L}_{1},\left[\mathrm{P}_{1}\right]_{\mathrm{FU}}$ and $\left.\left[\hat{P}_{1}\right]_{\mathrm{FU}}\right]$ when $\mathrm{p}=0.4, \mathrm{~L}_{2}=0.7, \mathrm{p}_{\mathrm{A}}=0.7$

| Fixed <br> parameter | $\mathrm{L}_{1}$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | COD |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{p}=0.4$ |  |  |  |  |  |  |  |  |  |  |  |
| $\mathrm{~L}_{2}=0.7$ | $\mathrm{P}_{1}$ | 0.3643 | 0.3278 | 0.2904 | 0.2521 | 0.2127 | 0.1724 | 0.1310 | 0.0884 | 0.0448 |  |
| $\mathrm{p}_{\mathrm{A}}=0.7$ | $\hat{P}_{1}$ | 0.3690 | 0.3291 | 0.2891 | 0.2492 | 0.2093 | 0.1694 | 0.1295 | 0.0896 | 0.0497 | $\mathbf{0 . 9 9 9 2}$ |

$\hat{a}=0.4089 ; \quad \hat{b}=-0.3991 ; \quad \hat{P}_{1}=0.4089-0.3991 .\left(L_{1}\right)$

Where $\mathbf{P}_{\mathbf{1}}=\left[\mathbf{P}_{\mathbf{1}}\right]_{\text {PIU }}$ and $\hat{P}_{1}=\left[\stackrel{\hat{P_{1}}}{1}\right]_{P I U}$

Table 9.4: [Values of $\mathrm{L}_{1},\left[\mathrm{P}_{1}\right]_{\text {PIU }}$ and $\left.\left[\hat{P}_{1}\right]_{\text {PIU }}\right]$ when $\mathrm{p}=0.4, \mathrm{~L}_{2}=0.3, \mathrm{p}_{\mathrm{A}}=0.2$

| Fixed <br> parameter | $\mathrm{L}_{1}$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | COD |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{p}=0.4$ <br> $\mathrm{~L}_{2}=0.3$ <br> $\mathrm{p}_{\mathrm{A}}=0.2$ | $\mathrm{P}_{1}$ | 0.4408 | 0.3945 | 0.3476 | 0.3001 | 0.2519 | 0.2029 | 0.1533 | 0.1029 | 0.0518 |  |
|  | $\hat{P}_{1}$ | 0.4440 | 0.3954 | 0.3468 | 0.2981 | 0.2495 | 0.2009 | 0.1523 | 0.1037 | 0.0551 | $\mathbf{0 . 9 9 9 7}$ |

$$
\begin{equation*}
\hat{a}=0.4926 ; \quad \hat{b}=-0.4860 ; \quad \hat{P}_{1}=0.4926-0.4860 .\left(L_{1}\right) \tag{9.6.4}
\end{equation*}
$$

Table 9. 5: $4\left[\right.$ Values of $\mathrm{L}_{1},\left[\mathrm{P}_{1}\right]_{\text {PIU }}$ and $\left.\left[\hat{P}_{1}\right]_{\text {PIU }}\right]$ when $\mathrm{p}=0.4, \mathrm{~L}_{2}=0.5, \mathrm{p}_{\mathrm{A}}=0.5$

| Fixed <br> parameter | $\mathrm{L}_{1}$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | COD |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{p}=0.4$ <br> $\mathrm{~L}_{2}=0.5$ <br> $\mathrm{p}_{\mathrm{A}}=0.5$ | $\mathrm{P}_{1}$ | 0.4429 | 0.3955 | 0.3476 | 0.2993 | 0.2506 | 0.2014 | 0.1517 | 0.1016 | 0.0510 |  |
|  | $\hat{P}_{1}$ | 0.4450 | 0.3960 | 0.3471 | 0.2981 | 0.2491 | 0.2001 | 0.1511 | 0.1021 | 0.0531 | $\mathbf{0 . 9 9 9 9}$ |

$\hat{a}=0.4940 ; \quad \hat{b}=-0.4898 ; \quad \hat{P}_{1}=0.4940-0.4898 .\left(L_{1}\right)$

Table 9.6: $4\left[\right.$ Values of $\mathrm{L}_{1},\left[\mathrm{P}_{1}\right]_{\mathrm{PIU}}$ and $\left.\left[\hat{P}_{1}\right]_{\mathrm{PIU}}\right]$ when $\mathrm{p}=0.4, \mathrm{~L}_{2}=0.7, \mathrm{p}_{\mathrm{A}}=0.7$

| Fixed <br> parameter | $\mathrm{L}_{1}$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | COD |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{p}=0.4$ <br> $\mathrm{~L}_{2}=0.7$ <br> $\mathrm{p}_{\mathrm{A}}=0.7$ | $\mathrm{P}_{1}$ | 0.4525 | 0.4038 | 0.3548 | 0.3054 | 0.2555 | 0.2052 | 0.1546 | 0.1034 | 0.0519 |  |
|  | $\hat{P}_{1}$ | 0.4544 | 0.4043 | 0.3543 | 0.3042 | 0.2541 | 0.2041 | 0.1540 | 0.1039 | 0.0539 | $\mathbf{0 . 9 9 9 9}$ |

$\hat{a}=0.5045 ; \quad \hat{b}=-0.5006 ; \quad \hat{P}_{1}=0.5045-0.5006 .\left(L_{1}\right)$

## X. CONFIDENCE OF INTERVALS (COI)

The $100(1-\alpha)$ percent confidence interval for $a$ and $b$ are:

$$
\begin{equation*}
\hat{a} \pm\left\{t_{(n-2)} \frac{\alpha}{2}\right\} \cdot s\left[\sqrt{\frac{1}{n}+\frac{\bar{L}_{1}}{\sum_{i=1}^{n}\left(L_{1 i}-\overline{L_{1}}\right)^{2}}}\right] \cdots \tag{10.1}
\end{equation*}
$$

Where $\overline{L_{1}}=\frac{1}{n} \sum_{i=0}^{n} L_{1 i}$. The $\overline{L_{1}}=4.5$ for all table (9.1, 9.2, 9.3, 9.4, 9.5 and 9.6 )
$\hat{b} \pm\left\{t_{(n-2)}, \frac{\alpha}{2}\right\} \cdot S\left[\sqrt{\sum_{i=1}^{n}\left(L_{1 i}-\overline{L_{1}}\right)^{2}}\right]$
Where $\mathrm{s}=\sqrt{\frac{\sum\left(P_{i}-\hat{P}_{i}\right)^{2}}{n-2}}$ and $t_{(n-2)} \frac{\alpha}{2}$ is obtained from standard table. Take $\alpha=0.05, \mathrm{n}=9$ then $\mathrm{t}_{7}, 0.025=2.365$

Table 10.1: Confidence interval for a and b using (9.5)

| Fixed parameter | Constant (a) | Constant $(\mathrm{b})$ | Confidence Interval |
| :---: | :---: | :---: | :---: |
| $\mathrm{p}=0.4, \mathrm{~L}_{2}=0.3, \mathrm{p}_{\mathrm{A}}=0.2$ | $\hat{a}=0.4242$ | $\hat{b}=-0.3922$ | For $\mathrm{a}:(\mathrm{a}=0.4012, \mathrm{a}=0.4472)$ <br> For $\mathrm{b}:(\mathrm{b}=-0.3739, \mathrm{~b}=-0.4105)$ |
| $\mathrm{p}=0.4, \mathrm{~L}_{2}=0.5, \mathrm{p}_{\mathrm{A}}=0.5$ | $\hat{a}=0.4150$ | $\hat{b}=-0.3973$ | For a: $(\mathrm{a}=0.4015, \mathrm{a}=0.4285)$ |
| $\mathrm{p}=0.4, \mathrm{~L}_{2}=0.7, \mathrm{p}_{\mathrm{A}}=0.7$ | $\hat{a}=0.4089$ | For $\mathrm{b}:(\mathrm{b}=-0.3866, \mathrm{~b}=-0.4081)$ |  |
|  | $\hat{b}=-0.3991$ | For a: $(\mathrm{a}=0.4011, \mathrm{a}=0.4167)$ |  |
| Average Estimates | $\bar{a}=0.4160$ | $\bar{b}=-0.3962$ | For b: $(\mathrm{b}=-0.3929, \mathrm{~b}=-0.4053)$ |
|  |  |  | $\hat{P}_{1}=\bar{a}+\bar{b}\left(L_{1}\right)$ |

Table 10.2: Confidence interval for $a$ and $b$ (9.6)

| Fixed parameter | Constant (a) | Constant (b) | Confidence Interval |
| :---: | :---: | :---: | :---: |
| $\mathrm{p}=0.4, \mathrm{~L}_{2}=0.3, \mathrm{p}_{\mathrm{A}}=0.2$ | $\hat{a}=0.4926$ | $\hat{b}=-0.4860$ | For $\mathrm{a}:(\mathrm{a}=0.4873, \mathrm{a}=0.4979)$ <br> For $\mathrm{b}:(\mathrm{b}=-0.4818, \mathrm{~b}=-0.4903)$ |
| $\mathrm{p}=0.4, \mathrm{~L}_{2}=0.5, \mathrm{p}_{\mathrm{A}}=0.5$ | $\hat{a}=0.4940$ | $\hat{b}=-0.4898$ | For $\mathrm{a}:(\mathrm{a}=0.4906, \mathrm{a}=0.4975)$ |
|  | For $\mathrm{b}:(\mathrm{b}=-0.4871, \mathrm{~b}=-0.4926)$ |  |  |
| $\mathrm{p}=0.4, \mathrm{~L}_{2}=0.7, \mathrm{p}_{\mathrm{A}}=0.7$ | $\hat{a}=0.5045$ | $\hat{b}=-0.5006$ | For $\mathrm{a}:(\mathrm{a}=0.5013, \mathrm{a}=0.5076)$ |
|  |  |  | For $\mathrm{b}:(\mathrm{b}=-0.4981, \mathrm{~b}=-0.5031)$ |
| Average Estimates | $\bar{a}=0.4970$ | $\bar{b}=-0.4922$ | $\hat{P_{1}}=\bar{a}+\bar{b}\left(L_{1}\right)$ |
|  |  |  | $\hat{P_{1}}=(0.4970-0.4922)\left(L_{1}\right)$ |

## XI. AVERAGE RELATIONSHIP

We define $\hat{P}_{1}=\bar{a}+\bar{b}\left(L_{1}\right) \quad$ in table 10.1 and 10.2 where $\bar{a}, \bar{b}$ are average estimate obtain through all tables. We found that $\hat{P}_{1}=0.4160-0.3962 .\left(L_{1}\right)$ and
$\hat{P}_{1}=0.4970-0.4922 .\left(L_{1}\right)$

## XII. DISCUSSION AND CONCLUSION

A linear relationship has been established between blocking probability and traffic sharing in the rest state setup. The equation on line depends on input parameters. The confidence intervals are very small length showing the strength of estimation procedure. The coefficient of variations is near to unity showing the high efficiency of line fitting procedure. The average linear relationship is
$\hat{P}_{1}=0.4160-0.3962\left(L_{1}\right)$
and $\hat{P}_{1}=\left(0.4970-0.4922\left(L_{1}\right)\right.$. This can be used for quick calculation of traffic sharing when blocking level of network varies for an operator.

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