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Scramble PAPR Reduction Algorithms for Fiber Nonlinearity Mitigation in Long Haul Coherent Optical OFDM Systems

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***Abstract*— Orthogonal Frequency Division Multiplexing (OFDM) technique was proved as an effective solution for mitigating chromatic dispersion (CD) and polarization-mode dispersion (PMD) effects in long haul optical communication systems. However, the large peak-to-average power ratio (PAPR) problem increases the presence of nonlinear effects, which are proportional to the instantaneous signal power. In this paper, we study selected mapping (SLM) and partial transmit sequence (PTS) technique to reduce peak-to-average power ratio. It is also known as a kind of scrambling technique. They are considered as two better methods because of their distortionless characteristics. The simulation results are also presented in this paper.**

***Index Terms* – OFDM, CO-OFDM, PAPR, SLM and PTS**

I. INTRODUCTION

OPTICAL communication has been advanced to deliver the highest bit rates ever imagined, up to several hundred Gbits/s per optical wavelength channel [1], [2]. This is possible due to the significant progresses in the use of coherent detection, orthogonal frequency division multiplexing (OFDM) technique, multiplexing of polarization modes of guided optical waves in single mode optical fibers, and the employment of ultra-high speed processing in the electronic domain. OFDM technique has been demonstrated to combat fiber impairments such as fiber chromatic dispersion (CD) and polarization-mode dispersion (PMD) by splitting one high data rate stream into many lower data rate streams and then modulating each of them on corresponding subcarriers. Thus, OFDM can tolerate inter-symbol interference (ISI) caused by fiber chromatic dispersion [3], therefore, it seems quite a potential technique in high data rate optical communication. However, this technique exist some drawbacks. One of them is a high PAPR since many subcarrier components are added via IFFT operation. The high PAPR gives rise to signal impairments which are caused by nonlinear devices' characteristics such as Analog/Digital (A/D) converter, Mach-Zehnder Modulator (MZM) as well as fiber cable [4]. In addition, the Kerr effect also makes distortions known as four-wave mixing (FWM) phenomenon between OFDM subcarriers. It makes subcarriers become dis-orthogonal. It is worth noticing that theinfluences of these nonlinear phenomena depend on the signal power which is measured before launching into fiber [5]. Therefore, various PAPR reduction techniques have been researched and proposed in wireless communications and recently for optical OFDM systems [3], [6], [8]. In wireless communication field, researchers already carried out many PAPR reduction methods such as filtering, clipping, coding, partial transmission sequences (PTS), selected mapping (SLM), etc [4], [9]. Among these methods, the SLM and PTS schemes are considered as more efficient for PAPR reduction algorithms. The idea is to scramble an input data block of the OFDM symbols in frequency domain (SLM) or in time domain (PTS) and multiply them by a set of phase factor. Finally, the one with the minimum PAPR is transmitted.

In our work we use both SLM and PTS reduction methods applying to optical communication employing OFDM technique to reduce fiber nonlinear effects. This paper is divided into five parts. After a short introduction, the fundamental PAPR theory in CO-OFDM systems is revealed in section two. In section three, the SLM and PTS algorithmsare illustrated in detail, CO-OFDM system set-up with these algorithmsare discussed and showed in next part. The last one is some of the numerical simulation results and discussion.

II. PAPR OF THE OFDM SIGNAL

In OFDM system with N subcarriers, if M signal are added with the same phase, they produce a peak power that is M times the average power. The complex baseband representation of an OFDM signal is expressed as [8]:

$$x(t) = \frac{1}{\sqrt{N}}\sum_{n=1}^{N} a_n \exp(j\omega_n t) \qquad (1)$$

Where a_n are the modulating symbols and ω_n are the carriers.

The PAPR of the signal is defined[8]:

$$PAPR(dB) = 10log_{10}\frac{max\{|x_t|\}^2}{E\{|x_t|^2\}} \qquad (2)$$

$E\{|x_t|^2\}$is the average power of OFDM signal [9].

The PAPR has the worst case value $PAPR_{WC}$ which depends on the number of subscribers N. This can be expressed in Table I and and Fig. 1. It can be written as [10]:

$$PAPR(dB) = 10log_{10}N \quad (3)$$

Table 1. $PAPR_{WC}$ vs Number of Subcarriers

No. of subcarrier	2	4	8	16	32	64	128	256
PAPRwc dB	3.01	6.02	9.03	12.04	15.05	18.06	21.07	24.01

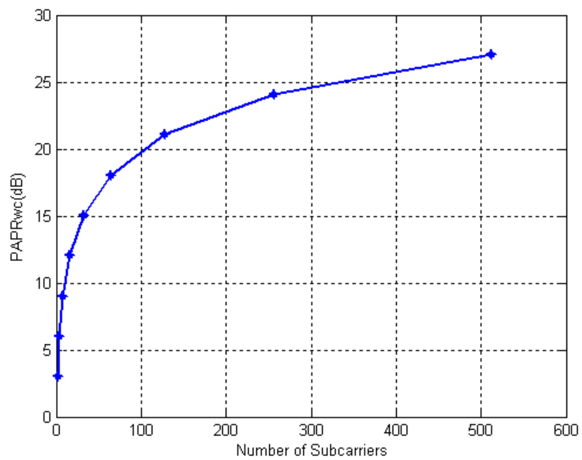

Figure 1. $PAPR_{WC}$ versus number of subcarriers

It can be easily seen from Table I and Fig. 1 that the PAPR problem is more and more serious as number of subcarriers increases.

The performance of PAPR reduction algorithms could be evaluated in the following ways: (1) In-band ripple or out of band radiation which can be seen through power spectral density, (2) distribution of PAPR which is given by complementary cumulative distribution function (CCDF), and (3) is coded and un-coded BER performance. In our work, we use (2) and (3) to evaluate system's performance since SLM is a distortionless PAPR reduction method. The formula of CCDF is as follows [8]:

$$P\{PAPR > z\} = 1 - P\{PAPR \le z\}$$
$$= 1 - (1 - e^{-z})^N \quad (4)$$

III. SCRAMABLE PAPR REDUCTION METHODS

A. SLM algorithm illustration

SLM scheme is one of the most efficient approaches in all of PAPR reduction methods. The OFDM signal is optimized before it is launched into fiber. This is done by combining different signal sub-blocks which are multiplied by a set of phase weighting factor to produce alternative transmit signal containing the same information. However, when we have a large number of sub-blocks, finding out the best weighting factor is a complex and difficult problem.

The block scheme of SLM is shown in Fig. 2. The complex input data **X** = [X[0], X[1],…, X[N-1]] is input to *U* scramblers, and then is multiplied with a set of phase sequence, result $\mathbf{X^u}$:

$$X^u = X.P^u \quad (5)$$

With P^u is the *u*-th scramble matrix:

$$P^u = \text{diag}[e^{j\phi q_0^u}, e^{j\phi q_1^u}, \dots, e^{j\phi q_{N-1}^u}] \quad (6)$$

$\phi \in [0,2\pi)$ is rotation phase, and $q_0^u \in \{0,1\}$

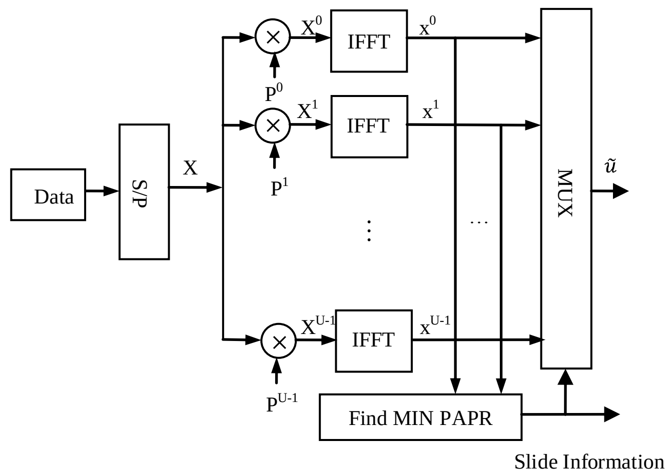

Figure 2. The structure of transmitter with SLM scheme

After multiplied with phase weighting factor, *U* sequences X^u are transformed into time domain by IFFT algorithm. Among them, SLM algorithm will select the sequence which has lowest PAPR value for transmission:

$$\tilde{u} = argmin_{u=1,2,\dots,U}\left(max_{n=0,1,\dots,N-1}|x^u[n]|\right) \quad (7)$$

At the receiver, to recover the original data stream, the side information (SI) related to the selected phase weighting sequence P^u should be used. So, SI must be transmitted for taking the data stream back. We can see that the SLM technique suffers from the complexity of finding the optimum set of weighting phase factor, especially when the number of sub-block is large.

Fig. 3 is SLM algorithm flowchart which describes how to build the "Find MIN PAPR" subsystem. We can see that the SLM technique suffers from the complexity of finding the optimum set of weighting phase factor, especially when the number of sub-block is large.

In our simulation, we divide the parallel complex data stream into four parts (sub-blocks). Each of them are then multiplied with a possible set of phase factor (1, -1, j, -j).

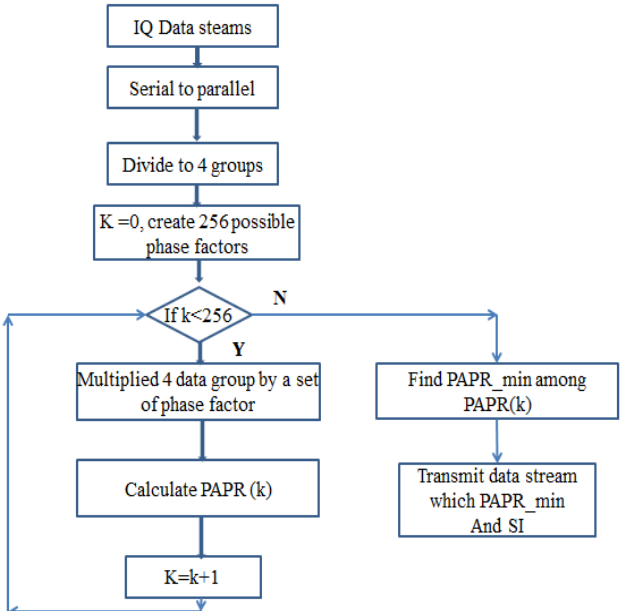

Figure 3. SLM algorithm flowchart

B. PTS algorithm illustration

PTS is also the most efficient approach and a distortionless scheme for PAPR reduction by optimally combining signal sub-blocks. In PTS technique, the input data block is broken up into disjoint sub-blocks in time domain. The sub-blocks are transformed into frequency domain by using IFFT, and after that they are weighted by a phase weighting factorbefore adding together to produce alternative transmit containing the same information (Fig. 4). However, when we have a large number of sub-blocks, finding out a best weighting factor is a complex and difficult problem [11].

In the same manner with SLM algorithm, the input data vector X in PTS algorithm is firstly partitioned into M disjointed sub-blocks$X_m = [X_{m,0}, X_{m,1}, \dots X_{m,N-1}]^T$such that:

$$\sum_{m=0}^{M-1} X^m = X, 0 \le m \le M-1 \tag{8}$$

The sub-blocks are combined to minimize the PAPR. After performing the Inverse Fast Fourier Transform of X_m, we have$x_m = [x_{m,0}, x_{m,1}, \dots x_{m,NL-1}]^T$, $0 \le m \le M-1$ with L is oversampling factor.

Each sub-block in time domainafter that is rotated by a phase factor set$b_m = e^{j\phi_m}$. In general, the phase factor set is limited with a finite number of elements to reduce the complexity. In this paper, we chose $\phi \in \{0, \frac{\pi}{2}, \pi, 3\pi/2\}$, this means $b_m \in \{\pm j, \pm 1\}$. Finally, the sub-blocks are summed up. After the PTS operation, the OFDM signal becomes [12].

$$x_{out} = \sum_{m=1}^{M} b_m . x_m \tag{9}$$

Where x and $x^{(m)}$ are the signal in the time domain.

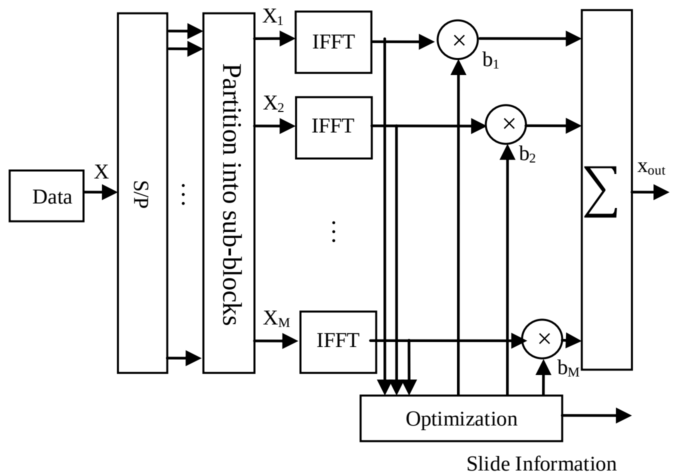

Figure 4. The structure of transmitter site with PTS

A phase factor set is created by Optimization sub-system. This is the flowchart of Optimization (Fig. 5):

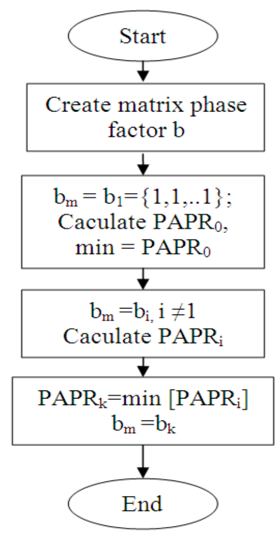

Figure 5: Flowchart of Optimization sub-system

The aim in the PTS is to find the optimal phase factors. In the phase optimization, because the phase factor of the first sub-block is taken as $b_0 = 1$, there are W^{M-1} alternative b combinations, where $b = [b_1, b_2, \dots, b_{M-1}]$ and W is the number of the phase factors. In sequence b, b_m values are as follows:

$$b_m = \begin{cases} \{\pm 1\} & if\ W = 2 \\ \{\pm 1, \pm j\} & if\ W = 4 \end{cases} \tag{10}$$

Therefore, the side information (SI) consists of the length of the SI is $R = (M-1)\log_2(W)$ bits. Fig. 6 illustrates an example of PTS method for OFDM system. It consists of 8 subcarriers which are divided into 4 sub-blocks. The phase factors are just $\{\pm 1\}$.

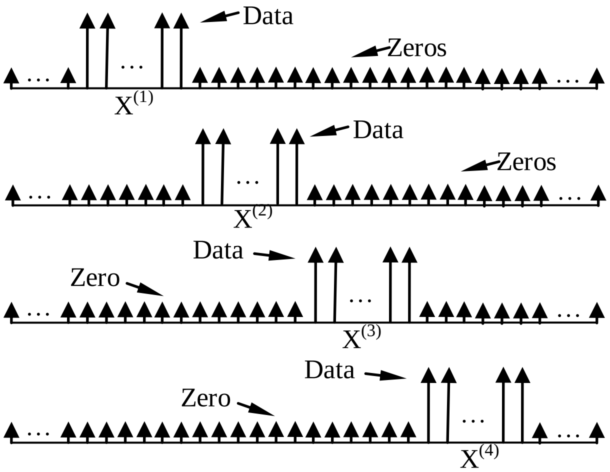

Figure 6. PTS 4 sub-blocks

IV. LONG-HAUL OPTICS FIBER COMMUNICATION SETUP

A single fiber transmission span consists of a Single Mode Fiber (SMF), an optical amplifier EDFA (Fig. 7).

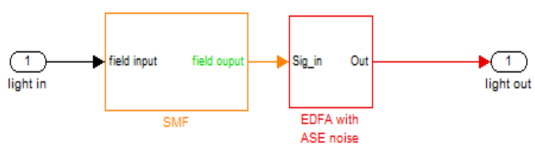

Figure 7. Single fiber transmission span

We simulate an optical communications link over several hundred kilometers by cascading these spans from one end of the transmission link to another. The loss of each span is compensated by an EDFA.

A. SMF modeling

The simulation of the optical signal which is propagated is based on the solution of the nonlinear Schrödinger equation (NES) [13], [14].

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A - \beta_1\frac{\partial A}{dt} - \frac{j}{2}\beta_2\frac{\partial^2 A}{dt^2} + \frac{1}{6}\beta_3\frac{\partial^3 A}{dt^3} - j\gamma|A|^2 A \qquad (11)$$

Where β_1 correspond to the various dispersion components of the fiber; β_2, β_3 are the chromatic dispersion parameters respectively; Losses over the fiber are considered through the attenuation α parameter, and fiber non-linearity are showed by the γ term.

The NES is regarded as the propagation equation of an optical pulse in single mode fiber. The numerical approach which is used to figure out the nonlinear Schrodinger equation is known as the Split-Step Fourier Method (SSFM). The accuracy and efficiency of this method depend on the distribution of step sizes along fiber and on both time and frequency domain resolutions. Finding an optimal step is not easy and depends on particular optical system. It is beyond our study. The accuracy could be improved among total number of steps. To be practical, the step size we choose in the simulation is 100 meters in each span which is 80 km long. The long haul fiber communication link in this simulation is simulated by cascading many single spans. The Fig. 8 is Simulink model of a 800 km fiber long which is formed from 10 single spans.

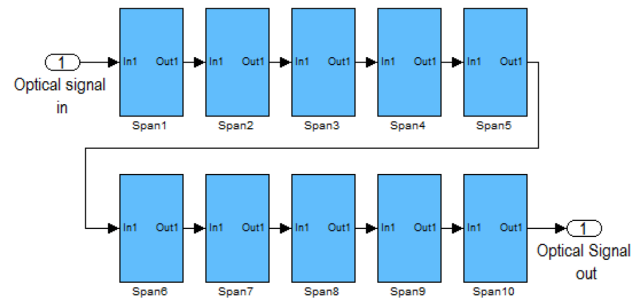

Figure 8. A 800 km fiber transmission link

The parameters of a single fiber link and EDFA are shown in Table 2.

Table 2.Fiber and EDFA parameters for single span

SMF	EDFA
Loss factor α = 0.2 dB/km Dispersion coeff. D = 17 (ps/nm.km) Nonlinear coeff. γ = 1.4e-4($m^{-1}.W^{-1}$) L = 80 km	G_dB = 16(dB) NF = 5

B. Cyclic Prefix

To tolerate the CD, a sufficient CP is suggested. It is the last part of OFDM symbol and inserting to the beginning of OFDM symbol. CP interval must be chosen [15]:

$$\Delta_G \geq \frac{c|D_t|N_{SC}}{f^2 t_s} \qquad (12)$$

Where f is frequency of optical carrier, c is speed of light, D_t is the total CD (ps/nm), N_{SC} is the number of OFDM subcarriers, and t_s is the OFDM symbol period.

C. CD cancelation

At the receiver, CD can be evaluated and canceled by using those equations [15]:

$$\Phi = \frac{1}{2}\beta_2\omega^2 L \qquad (13)$$

$$\text{Where } \beta_2 = -\frac{\lambda^2}{2\pi c}D$$

D is fiber dispersion, β_2 is group velocity dispersion, ω is the optical frequency at each subcarriers.

D. CO-OFDM System simulation model

The block diagram of CO-OFDM is shown in Fig. 9. A very high speed data is firstly modulated by using 4-QAM. After serial to parallel conversion, IFFT algorithm is performed to convert signal from frequency domain to time domain. They are then added CP, performed DAC converter and finally converted to optical domain via external modulation MZM.

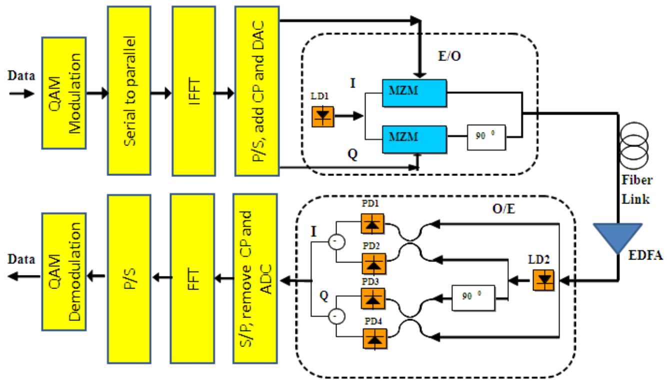

Figure 9. CO-OFDM system

At the receiver, after converting the signal from optical to electrical domain by using photo-detectors, the electrical signal is processed to give back the data via de-OFDM modulation and 4-QAM de-modulation.

In our simulation, the data rate is simulated at 10 Gbps and we have totally 256 subcarriers. It means that we use 256 points IFFT/FFT transformation.

V. EXPERIMENTAL RESULTS

This part illustrates the results of Simulink simulations conducted to evaluate the performance of long-haul CO-OFDM system with and without SLM and PTS algorithm.

A. SLM and PTS efficiency

The PAPR reduction performance of SLM and PTS algorithms are evaluated by the CCDF. Fig. 10 and Fig. 11 show the comparison of PAPR performance in term of CCDF of SLM and PTS.

If the number of sub-block is larger than 3, the performance of this algorithm is almost the same, around 3dB better than OFDM system without SLM at 5.10^{-2} CCDF. So, 4 sub-block case is the best choice. Fig. 10 is a rotation phase dependence of SLM algorithm via CCDF in case of 4 sub-blocks.

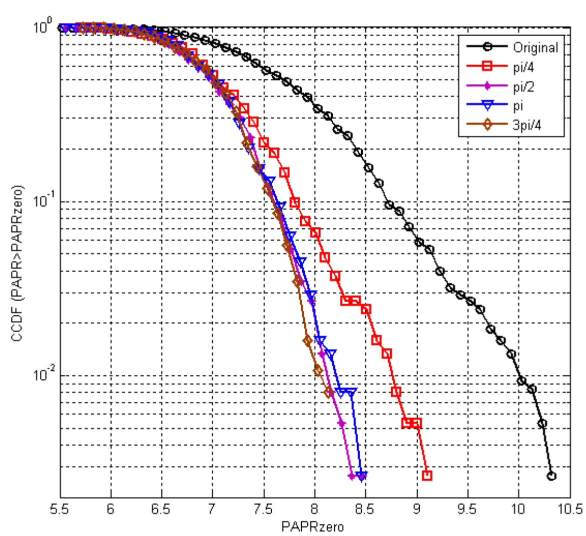

Figure 10. Rotation phase dependence of SLM incase of 4 sub-blocks

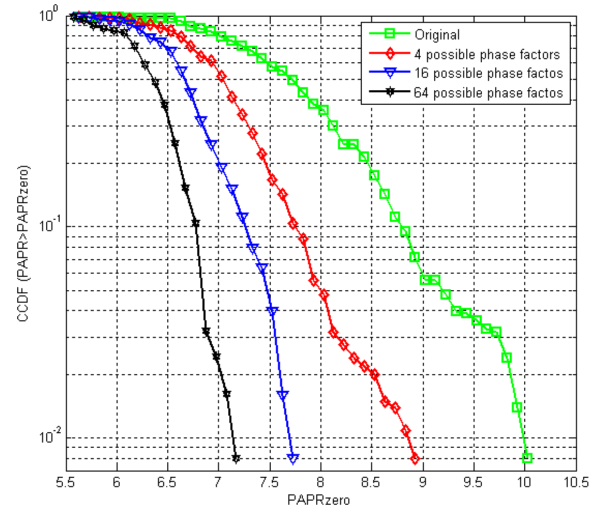

Figure 11. CCDF comparison of PTS with different number of phase factors

As can be seen from Fig.10, a good choice is obtained when $\phi > 3\pi/4$. From these results, we chose $\phi = \pi$ for the QPSK mapping. Fig.11 reveals the performance of PTS in term of number phase factor. We can see that the PTS even gives the better result when the number of combination phase factor increases. However, the system complexity becomes a big trouble as number of phase factor is large. From Fig.11, we chose the number of combination at 16. It gave quite good performance as well as reasonable complexity.

B. Long haul optical communication link experiment results

The system is demonstrated for a transmission up to 1000 km of standard-single-mode-fiber (SSMF) without dispersion compensation at 10Gb/s. The tolerance of the models to nonlinear effects is tested by increasing the average launched power into the fiber. The nonlinear threshold which is used in this model is 10mW.

In both Fig.12 and Fig.13, we can see that at low launched power, both systems with and without PAPR reduction algorithms have similar performances. At 3 dBm in SLM and 2.5 dBm in PTS, the performance of CO-OFDM with and without reduction methods is almost the same. The quality of system could be acceptable for around 1000 km fiber long. BER of both systems is still below 10^{-9} for such a long haul optics communication link.

When we increase the launched powers, system performance is now influenced by nonlinear effects. Therefore, the efficiency of SLM and PTS is represented clearly. As we can see in both Fig. 12 and Fig. 13, the performance of both systems with PAPR reduction algorithms is better than systems without these algorithms in nonlinear region. Specifically, we gain appropriately 50 km longer than system without SLM in case of launched power level 4 and 6 dBm (Fig. 12). This number is even span to about 200 km with PTS algorithm (Fig. 13). Compare with PTS in Fig. 13, we can conclude that PTS is quite better than SLM at the same input power level. In two consideration regions at Fig. 12 and Fig. 13, we can easy see that PTS gives the wider distance than SLM do.

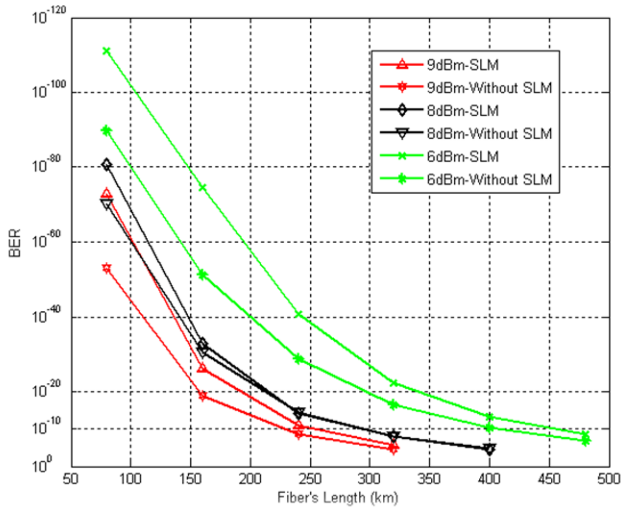

Figure 12. Performance of long haul optics fiber link with SLM

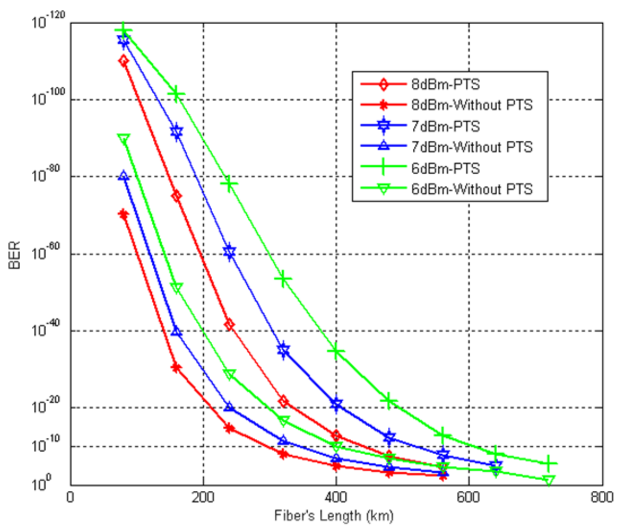

Figure 13. BER of long haul optics fiber link with PTS algorithm

VI. CONCLUSION

OFDM technique is a very attractive approach for long haul high speed optical transmission system. However, the PAPR problem is one of the important aspects needed to consider. In this article, we have fundamentally simulated two scrambling algorithms for PAPR reduction purpose, namely SLM and PTS applying for point-to-point long haul coherent optical – CO-OFDM system. As a result, system tolerance of nonlinear effects increases with these algorithms. However, PTS could be a better choice due to higher performance. It is necessary to study some algorithms for reducing the complexity of the both PAPR reduction methodsinoptical OFDM communication.

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