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# Recognition of Degraded Images by Legendre Moment Invariants

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**Abstract**– Analysis and interpretation of an image which was acquired by a non ideal imaging system is the key problem in many application areas. The observed image is usually corrupted by blurring, spatial degradations, and random noise. In this paper, we propose an alternative approach. We derive the features for image representation which are invariant with respect to blur regardless of the degradation point spread function (PSF) provided that it is centrally symmetric. Methods to obtain blur invariants which are invariants with respect to centrally symmetric blur are based on geometric moments or complex moments, orthogonal legendre moments. The performance of the proposed descriptors is evaluated with various point-spread functions and different image noises. The comparison of the different approaches with previous methods in terms of pattern recognition accuracy is also provided.

**Index Terms**– Blurred Image, Centrally Symmetric, Legendre Moments, Pattern Recognition and Symmetric Blur

## I. INTRODUCTION

**B**LURRING due to object or camera motion during image capture can cause substantial degradation in image quality. As a result, a great deal of research has been conducted on developing methods for restoring motion blurred images. These methods make certain assumptions on the blurring process, the ideal image, and the noise. Various image processing techniques are then used to identify the blur and restore the image.

In scene analysis, we often obtain the input information in a form of an image captured by a non ideal imaging system. Most real cameras and other sensors can be modeled as a linear space-invariant system, where the relationship between the input “ $f(x, y)$ ” and the acquired image “ $g(x, y)$ ” is described as

$$g(x, y) = a(f * h)(x, y) + n(x, y) \quad (1)$$

In the above model,  $h(x, y)$  is the point-spread function (PSF) of the system, “ $n(x, y)$ ” is an additive random noise,  $a$  is a constant describing the overall Change of contrast, “ $t$ ” stands for a transform of spatial coordinates due to projective imaging geometry and  $*$  denotes 2D convolution. In many application

Areas, it is desirable to find a description of the original scene that does not depend on the imaging system without any prior knowledge of its parameters [1] - [5]. Basically, there are two different approaches to this problem: image normalization or direct description by invariants [1] - [5]. Image normalization consists of two major steps: geometric registration, that eliminates the impact of imaging geometry and transforms the image into some standard form, and blind de-convolution, that removes or suppresses the blurring. Both these steps have been extensively studied in the literature.

In the invariant approach we look for image descriptors (features) that do not depend on “ $h(x, y)$ ”, “ $t(x, y)$ ” and “ $a$ ”. In this way we avoid a difficult inversion of Eq. (1). In many applications, the invariant approach is much more effective than the normalization. Typical examples are the recognition of objects in the scene against a database, template matching, etc. The pioneering work in this field was performed by Flusser and Suk [6] who derived invariants to convolution with an arbitrary Centro symmetric PSF. These invariants have been successfully used in template matching of satellite images, in pattern recognition [7] - [10] in blurred digit and character recognition [11], [12], in normalizing blurred images into canonical forms [13], [14] and in focus/defocus quantitative measurement.

The extension of blur invariants to  $N$  -dimensions has also been investigated [20], [21]. All the existing methods to derive the blur invariants are based on geometric moments or complex moments. However, both geometric moments and complex moments contain redundant information and are sensitive to noise especially when high-order moments are concerned. This is due to the fact that the kernel polynomials are not orthogonal. Teague has suggested the use of orthogonal moments to recover the image from moments [22]. It was shown that the orthogonal moments are better than other types of moments in terms of information redundancy, and are more robust to noise.

In this paper, a new method to derive a set of blur invariants based on orthogonal Legendre moments was proposed. The organization of this project is as follows the theory of blur invariants of geometric moments and the definition of Legendre moments. In the relationship between the Legendre moments of the blurred image and those of the original image and the PSF. Based on this relationship, a set of blur invariants using Legendre moments is provided, the experimental results for evaluating the performance of the proposed descriptors.

II. MOMENT INVARIANTS

An essential issue in the field of pattern analysis is the recognition of objects and characters regardless of their position, size and orientation as illustrated in Fig. 1. The idea of using moments in shape recognition gained prominence when Hu (1962), derived a set of invariants using algebraic invariants.

Two-dimensional moments of a digitally sampled M X M image that has gray function f (x, y), (x y=0, 1, 2...M- 1) is given as

$$m_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{M-1} x^p \cdot y^q f(x, y) \tag{2}$$

The moments f(x, y) translated by an amount (a, b) are defined as,

$$\mu_{pq} = \sum_x \sum_y (x + a)^p (y + b)^q \cdot f(x, y) \tag{3}$$

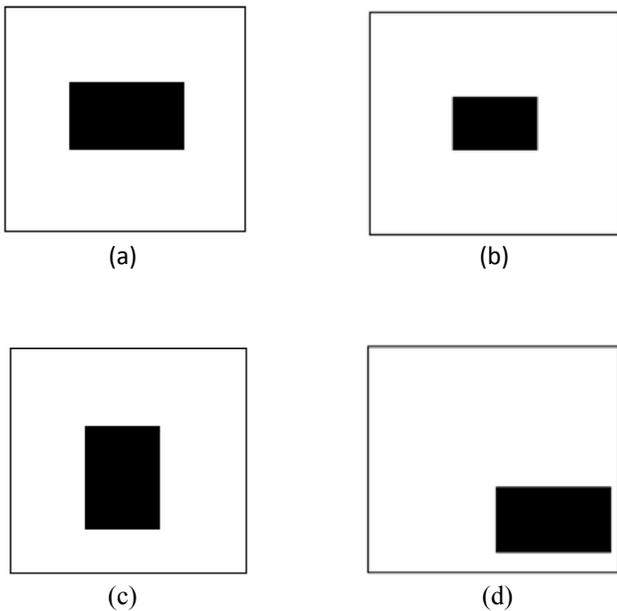


Fig. 1. (a) 2-d object (b) change of size (c) change of orientation (d) change of position

III. BASIC TERMS AND MATHEMATICAL FOUNDATION

First we define the basic terms which will be then used in construction of the invariants.

*Definition 3.1:* By image function (or image) we understand any real function f(x, y) having a bounded support and a finite nonzero integral.

*Definition 3.2:* Fourier transform (or spectrum) F (u, v) of the image f(x, y) is defined as:

$$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-2\pi i(ux+vy)} f(x, y) dx dy \tag{4}$$

Where i is the complex unit

*Definition 3.3:* Geometric moment “ $m_{pq}$ ” of image f(x; y), where p,q are non-negative

Integers and (p + q) is called the order of the moment, is defined as:

$$m_{pq}^{(f)} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q f(x, y) dx dy \tag{5}$$

Corresponding central moment  $\mu_{pq}$  of order (p+q) of this image f(x, y) is defined as

$$\mu_{pq}^{(f)} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x - x_t^{(f)})^p (y - y_t^{(f)})^q f(x, y) dx dy \tag{6}$$

Where the coordinates,

$$x_t^{(f)} = \frac{m_{10}^{(f)}}{m_{00}^{(f)}} \\ y_t^{(f)} = \frac{m_{01}^{(f)}}{m_{00}^{(f)}}$$

Where the coordinates  $(x_t^{(f)}, y_t^{(f)})$  denotes the Centriod

*Definition 3.4:* Complex moment “C (f)<sub>pq</sub>” of order (p + q) of the image f(x; y) is defined as

$$c_{pq}^{(f)} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x + iy)^p (x - iy)^q f(x, y) dx dy \tag{7}$$

Where i is the complex unit

*Definition 3.5:* h(x, y) is Central symmetry image function and the imaging system is energy preserving that is

$$h(x, y) = h(-x, -y) \tag{8}$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y) dx dy = 1 \tag{9}$$

Most real sensors and imaging systems have PSF s with certain degrees of symmetry. In many cases they have even higher symmetry than central, such as axial or radial symmetry. Thus, the Central symmetry assumption is general enough to describe almost all practical situations.

*Definition 3.6:* The Centriod the blurred image “g(x, y)” is related to Centriod of the original image “f(x, y)” and that of PSF “h(x,y)” as

$$x_t^{(g)} = x_0^{(f)} + x_0^{(h)} \\ y_t^{(g)} = y_0^{(f)} + y_0^{(h)} \tag{10}$$

In particular if h(x, y) is centrally symmetric then  $x_0^h = y_0^h = 0$  and in such case we have  $x_0^g = x_0^f, y_0^g = y_0^f$

#### IV. ORTHOGONAL MOMENTS

Cartesian moments, are formed using a monomial basis set  $x^p y^q$  this basis set is non-orthogonal and this Property is passed onto the Cartesian moments. These monomials increase rapidly in range as the order Increases, producing highly correlated descriptions. This can result in important descriptive information being contained within small differences between moments, which lead to the need for high computational precision.

However, moments produced using orthogonal basis sets exist. These orthogonal moments have the advantage of needing lower precision to represent differences to the same accuracy as the monomials. The Orthogonality condition simplifies the reconstruction of the original function from the generated moments. Orthogonality means mutually perpendicular, expressed mathematically two functions  $y^m$  and  $y^n$  are orthogonal over an interval  $a < x < b$  if and only if:

$$\int_a^b y_m(x)y_n(x)dx = 0; \quad m \neq n \quad (11)$$

Here we are primarily interested in discrete images, so the integrals within the moment descriptors are replaced by summations. It is noted that a sequence of polynomials which are orthogonal with respect to integration, are also orthogonal With respect to summation, one such (well established) orthogonal moment is Legendre.

#### V. LEGENDRE MOMENTS

The 2-D Legendre moments of order  $(p+q)$  of image function  $f(x,y)$  defined as

$$L_{pq}^{(f)} = \int_{-1}^1 \int_{-1}^1 P_p(x) P_q(y) f(x,y) dx dy \quad (12)$$

Where  $p, q=0, 1, 2, 3 \dots$ infinity.  $P_p, P_q$  Legendre polynomials and  $f(x,y)$  continuous image function. The Legendre polynomials are a complete orthogonal basis set defined over the interval  $[-1,1]$ , orthogonality to exist in the moments, the image function  $f(x,y)$  is defined over the same interval as the basis set, where the “n<sup>th</sup>” order Legendre polynomial is defined as:

$$P_p(x) = \sum_{k=0}^n c_{p,k} x^k \quad (13)$$

And the Legendre coefficients given by:

$$c_{p,k} = \begin{cases} \sqrt{\frac{2p+1}{2}} \frac{(-1)^{\frac{p-k}{2}} (p+k)!}{2^p (\frac{p-k}{2})! (\frac{p+k}{2})! k!} & p - k = \text{even} \\ 0 & p - k = \text{odd} \end{cases} \quad (14)$$

So, for a discrete image with current pixel  $P_{xy}$

$$\lambda_{pq} = \frac{(2m+1)(2n+1)}{4} \sum_x \sum_y P_p(x) P_q(y) P_{xy} \quad (15)$$

And  $x, y$  are defined over the interval  $[-1,1]$ .

The corresponding Central's moments are defined as 
$$\bar{L}_{pq}^{(f)} = \int_{-1}^1 \int_{-1}^1 P_p(x - x_0^{(f)}) P_q(y - y_0^{(f)}) f(x,y) dx dy \quad (16)$$

Where the coordinates  $(x_0^{(f)}, y_0^{(f)})$  denotes the Centriod

#### VI. EXPERMIENTS

##### A. Recognition Rates for Degraded Images by Moment Invariant

A toy image, whose size is 128 X128X 3 (Fig. 2) has been chosen from the public Columbia database. This image is degraded by average blur and salt and pepper noise. The parameter  $\sigma$ (standard deviation of Gaussian function)of average blur chosen As equal to 56.the other parameters such as salt and pepper noise is also added with noise density of 0.3.the original image was blurred by 3 X 3,4 X 4,5 X 5,6 X 6,7 X 7,8 x 8 9 X 9 10 x 10 averaging mask are shown in the Fig. 3.

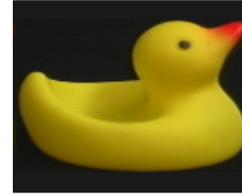


Fig. 2. Original image of size 128 X 128 X3

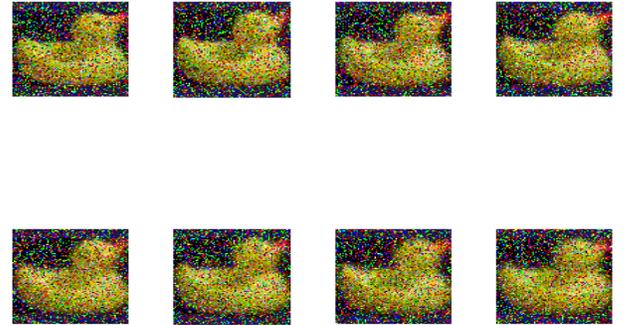


Fig. 3. Degrade image with average blur and salt and pepper Noise with varying mask sizes

The recognition rate for the above image by using geometric moments invariants and complex moment invariants is 35.56% &43.58%.the recognition rate for Legendre moment invariants is 95.81%,from this example we can say that Legendre moment invariants is best compared with complex and geometric moment invariants.

Some of the examples of images selected from image database of Columbia University are chosen the Fig. 4.

TABLE I: RECOGNITION RATES FOR DEGRADED IMAGES

Blur Type	Noise Type	Geometric moment invariant	Complex moment invariant	Legendre moment invariants
No blur	No noise	100%	100%	100%
Average	Gaussian noise with STD=8	91.61 %	89.46%	92.70%
Average	Gaussian noise with STD=16	86.49%	83.17 %	93.53%
Average	Gaussian noise with STD=20	82.91%	80.68%	89.90%
Average	Salt & pepper noise with density=0.01	98.67%	98.95%	98.91%
Average	Salt & pepper noise with density=0.02	98.27%	97.62%	99.13%
Average	Salt & pepper noise with density=0.03	96.23%	96.23%	98.90%
Average	multiplicative noise with STD=0.1	82.69%	80.26%	99.18%
Average	multiplicative noise with STD=0.2	71.38%	67.57%	95.96%
Average	multiplicative noise with STD=0.3	64.16%	60.59%	93.82%
Gaussian	Gaussian noise with STD=8	91.04%	88.67%	93.26%
Gaussian	Gaussian noise with STD=16	86.20 %	83.35%	95.71%
Gaussian	Gaussian noise with STD=20	82.78%	79.43 %	83.39%
Gaussian	Salt & pepper noise with STD=0.01	98.49%	99.29%	99.91%
Gaussian	Salt & pepper noise with STD=0.03	96.45%	96.63%	99.23%
Gaussian	multiplicative noise with STD=0.1	82.54%	80.03%	98.52%
Gaussian	multiplicative noise with STD=0.2	71.77%	67.33%	94.79%
Gaussian	multiplicative noise with STD=0.3	64.17%	61.07%	93.09%
motion	Gaussian noise with STD=8	91.26%	89.73%	95.68%
motion	Gaussian noise with STD=16	86.61%	84.73%	91.97%
motion	Gaussian noise with STD=20	82.82%	80.02%	80.83%
motion	Salt & pepper noise with STD=0.01	99.12%	98.33%	99.14%
motion	Salt & pepper noise with STD=0.02	97.32%	97.91%	98.98%
motion	Salt & pepper noise with STD=0.03	96.68%	96.03%	98.87%
motion	multiplicative noise with STD=0.1	82.76%	80.23%	99.10%
motion	multiplicative noise with STD=0.2	71.52%	67.73%	95.89%
motion	multiplicative noise with STD=0.3	63.78%	60.65%	94.35%



Fig. 4. Four color images selected from image data base of Columbia University



Fig. 5. Some examples of the degraded images with 3X3 mask

The recognition rates for the different moment invariants are given in the following table 1 for varying blur and noises. They lead to the same conclusions regarding the performance of the respective moment invariants but the decrease in recognition rate is more significant when the Noise level is increased. This is also true for the LMI. The CMI do not perform well in these experiments Table I.

*B. Performance Analysis of Moment Invariants*

A standard grey level image of size 128 X 128 X 3 is shown in the Fig. 2. This experiment was carried out to verify the performance of the invariants to both blur and noise. The original image was blurred by 9 X 9 averaging mask. Some examples of the blurred image with different types of noise are shown in Fig. 6. The programs were implemented in MATLAB 6.5 on a PC P4 2.4 GHZ, 512M RAM. It can be seen from Tables I and II that the GMI and the CMI, LMI performs better result in recognition of degraded images.

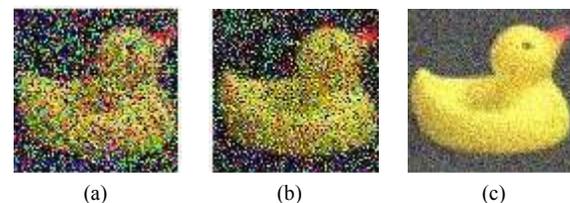


Fig. 6. Some examples of the degraded images: (a) Gaussian blur with salt and pepper noise (b) Average blur with salt and pepper noise(c) average blur with Gaussian noise

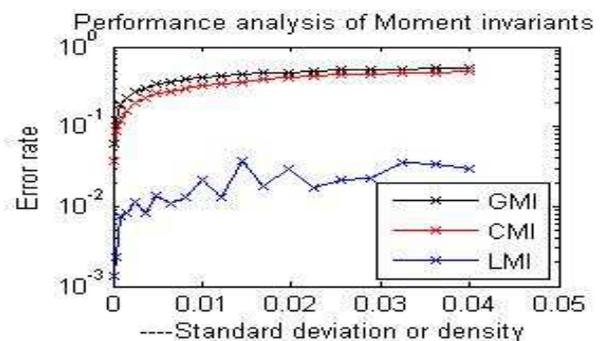


Fig. 7. Error rate for Gaussian blur and salt and pepper noise for Fig. 6 (a) Horizontal axis standard deviation or density and vertical axis error rate.

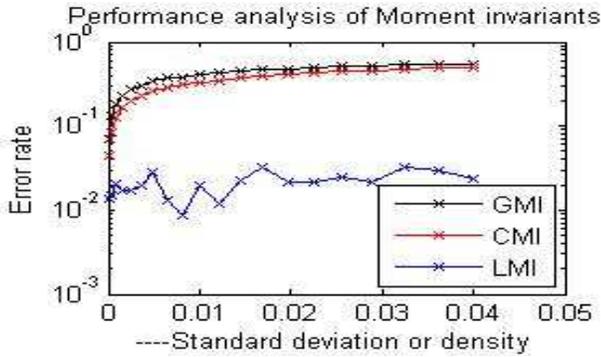


Fig. 8. Error rate for average blur and salt and pepper noise for Fig. 6 (b) Horizontal axis standard deviation or density and vertical axis error rate

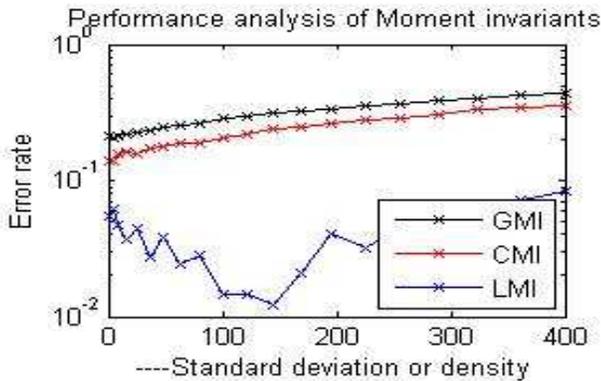


Fig. 9. Error rate for average blur and Gaussian noise for Fig. 6(c) Horizontal axis standard deviation or density and vertical axis error rate

From the above Fig. 7 – Fig. 9 it can be seen that LMI performs better than GMI, CMI. It can be also seen that a better robustness is achieved whatever the PSF or the additive noises for the original image by using LMI.

### C. Moment Invariants of Order up to Five for LMI, GMI and CMI

The moment invariants of original image shown in Fig. 2 up to order five is shown in the following Table II.

TABLE II: CALCULATION OF VARIOUS MOMENTS OF ORDER FIVE

MOMENTS OF ORDER	LMI	GMI	CMI
(5,0)	0	74.729	56.9078
(4,1)	42.36384	-46.81451	72.7031
(3,2)	-5.095288	17.41009	266.155
(2,3)	17.71161	-18.44362	243.8148
(1,4)	-8.146794	-28.73874	564.2339
(0,5)	0	264.9122	15.03025

## VII. CONCLUSION

The paper was devoted to the image features which are invariant to blurring by a filter with centrally symmetric PSF. In this paper, we have proposed a new approach to derive a set of blur invariants using the orthogonal Legendre moments. We demonstrated that invariant Functional can be used in image analysis as features for description and recognition of objects in

degraded images. Invariant-based approach is a significant step towards robust and reliable object recognition methods. The experiments conducted so far in very distinct situations demonstrated that the proposed descriptors are more robust to noise and have better discriminative power than the methods based on geometric or complex moments. The derivation of combined invariants to both geometric transformation and blur is currently under investigation.

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