Abstract—Cooperative Spectrum Sensing (CSS) has been developed to improve the detection performance of licensed Primary Users (PUs) in Cognitive Radio (CR) networks. The current study, introduces a novel weighted soft-decision combining scheme. The method is based on the estimation of instantaneous Signal-to-Noise Ratios (SNRs) of all CR users. An optimum threshold to minimize the total error probability is also provided. The proposed method is investigated through closed form expressions of total error probability. The numerical results are presented to verify the performance improvement of the proposed method compared with the conventional Equal Gain Combining (EGC) scheme.

Index Terms—Cooperative Spectrum Sensing, Soft-Decision Combining, Cognitive Radio and Equal Gain Combining

I. INTRODUCTION

COGNITIVE RADIO (CR) network is a new technology that opportunistically uses frequency spectrum [1]. In CR network, the CR users, which are referred as secondary users, can access to licensed primary band owned by the primary network [1], [2]. Spectrum sensing is a key technology to help the CR users to detect the vacant frequency bands [3]. Obviously, spectrum sensing individually performed by CR users often suffers from multipath fading, shadowing, and hidden station problems [4]. To mitigate these issues, Cooperative Spectrum Sensing (CSS) has been suggested [4], [5]. In CSS process, the sensing results of multiple CR users are combined to decide the existence of a Primary User (PU) signal. In cooperative sensing, each CR user sends its decision information over common control channel to the Fusion Center (FC) and an appropriate combining scheme is addressed in the FC to make the final decision about the presence or absence of the PU signal.

There are two different combining methods for the CSS, soft-combining and hard-combining schemes [6]. In soft-combining, the local measured energies by CR users are sent to the FC to make the global decision. In hard-combining, the local measured energies are compared with a predefined threshold and the binary decision symbols are sent to the FC. The soft-combining scheme shows a better sensing performance than hard-combining at the cost on increasing bandwidth of the common control channel [7].

Many cooperative sensing approaches have been investigated so far. A weighted cooperative sensing framework for a centralized CR network is studied in [8]. The local error probabilities and the measured energies from the PU signal are sent to the FC. The sensing information from a CR user that has a higher error probability is assigned with a lower weight. In [9], the authors propose consensus-based weight for energy detection in soft-combining scheme. In their proposed method, each CR user exchanges its own measurement data with its local one-hop neighbors and chooses the information exchanging rate according to the measurement SNR. In [10], a novel weighted energy detection scheme is proposed for a centralized CR network. The FC allocates an appropriate weight for each CR user according to the value of received energy. The allocated weight is based on the area under curve in Probability Distribution Function (PDF) curve of received data. In addition, an optimum threshold to minimize the total error probability is provided.

The current study, introduces a novel weighted energy detection approach based on soft-decision combining. Each CR user measures the received energy from PU signal and then sends its obtained value to the FC via an error free communication channel. The FC estimates the instantaneous Signal-to-Noise Ratio (SNR) of each user from received energy samples and allocates an appropriate dynamic weight to each CR user according to its instantaneous SNR. Moreover, an optimum threshold to minimize the total error probability is provided.

II. SYSTEM MODEL

We assume one PU transmitter, $N$ cooperative CR users, and one FC. The CR users are randomly deployed in a small area and the FC is located at the center of the CR network as depicted in Fig. 1.
It is assumed that the communication range of the PU transmitter covers the whole network. We also assume that energy detection scheme is used for local spectrum sensing. The measured energy of each CR user is sent to the FC and Rayleigh fading channel is considered between PU and each CR user. In addition, $M$ samples are utilized for local energy detection of each user and determined from time-bandwidth product [11]. Based on the presence or absence of the PU signal, the local spectrum sensing has two possible states which can be expressed as

$$
\begin{align*}
H_0 : & \text{ only Noise} \\
H_1 : & \text{ PU Noise}
\end{align*}
$$

The hypothesis $H_0$ occurs when the CR users receive only noise and the second state $H_1$ happens when the PU transmits over the channel. By considering two hypotheses $H_0$ and $H_1$, the received signal at the $i$ th sample of the $j$ th CR user, $x_j^i$, can be written as

$$
x_j^i = \begin{cases} 
n_j^i & H_0 \\
\sqrt{\gamma_j}s_j^i + n_j^i & H_1
\end{cases}
$$

where $n_j^i$ is the noise sample at the $j$ th CR user and $\sqrt{\gamma_j}s_j^i$ is the received PU signal with the power $\sqrt{\gamma_j}$. The noise sample $n_j^i$ and the PU signal sample $s_j^i$ are assumed to be independently and identically distributed Gaussian random variables with zero mean and unit variance. We further assume that the CR users experience independent Rayleigh fading sensing channels with the same average SNRs. This condition is relevant for the CR users which are randomly deployed in a small area and the network is geographically far from the PU transmitter. Thus, $\gamma_j$ varies from observation (period) to period while its PDF is identically as exponential distribution with the average value $\bar{\gamma}$.

$$
f(\gamma_j) = \frac{1}{\gamma_j}\exp(-\frac{\gamma_j}{\bar{\gamma}})
$$

With regard to the above assumptions, the received signal, $x_j^i$, is a Gaussian distributed as [6],

$$
x_j^i \sim \begin{cases} 
\mathcal{N}(0, 1) & H_0 \\
\mathcal{N}(0, \gamma_j + 1) & H_1
\end{cases}
$$

Moreover, each CR user utilizes $M$ samples for its local energy detection [11]. The obtained local energy of the $j$ th user, $E_j$, is given by

$$
E_j = \sum_{i=1}^{M} |x_j^i|^2 = \begin{cases} 
a_j & H_0 \\
(\gamma_j + 1)b_j & H_1
\end{cases}
$$

where the parameters $a_j$ and $b_j$ are two central Chi-square random variables with $M$ degrees of freedom. But, according to central limit theorem, if a large number of samples are utilized (i.e. $M > 10$), $E_j$ can be assumed to be Gaussian distributed as

$$
E_j \sim \begin{cases} 
\mathcal{N}(M, 2M) & H_0 \\
\mathcal{N}(M(\gamma_j + 1), 2M(\gamma_j + 1)^2) & H_1
\end{cases}
$$

In CSS process, the local measured energies are sent to the FC to make a final decision about presence or absence of the PU signal. In conventional EGC scheme [6], all of the sensing reports are summed up and compared with a predefined threshold. If the sum of reports is greater than the threshold then the frequency band is determined to be occupied; otherwise, the channel is assumed to be idle. The output signal at the FC is

$$
Y = \sum_{j=1}^{N} E_j > \lambda
$$

where $\lambda$ is the global threshold and determined by the target false alarm or miss detection probabilities. Obviously, the decision statistic $Y$ is also a Gaussian distributed random variable and can be defined as

$$
Y \sim \begin{cases} 
\mathcal{N}(\mu_0, \sigma_0^2) & H_0 \\
\mathcal{N}(\mu_1, \sigma_1^2) & H_1
\end{cases}
$$

where one can easily verify that

$$
\mu_0 = MN, \quad \sigma_0^2 = 2MN \\
\mu_1 = MN(\gamma + 1), \quad \sigma_1^2 = 2MN(\gamma + 1)^2
$$

The total detection, false alarm, and miss detection probabilities can be written in general as

$$
Q_d = p(0 > \lambda | H_0) = Q\left(\frac{\lambda - \mu_0}{\sigma_0}\right) \\
Q_f = p(0 > \lambda | H_1) = Q\left(\frac{\lambda - \mu_1}{\sigma_1}\right) \\
Q_m = p(0 < \lambda | H_1) = 1 - Q_d
$$

The total error probability can also be computed as

$$
Q_e = Q_f p(H_0) + Q_m p(H_1)
$$

where $Q(.)$ is a Q-function for standard normal distribution, $P(H_0)$ and $P(H_1)$ are the idle and busy rates of the channel, respectively.
The distribution functions of the decision statistics \( Y \), under two hypothesis \( H_0 \) and \( H_1 \) are shown in Figure 2, the values of \( Q_{fa} \) and \( Q_{m} \) are also presented.

Fig. 2. Conditional PDFs of the FC’s Decision Statistics

III. THE PROPOSED METHOD

As mentioned before, the received energies from all CR users are combined in FC and two important strategies are addressed to make a final decision on the channel status. At first, an appropriate weight is innovatively assigned for each CR user. Second, an optimal global threshold, in order to minimize the total error probability, is obtained at the FC.

A) An Appropriate Weight Allocation

The main contribution for weight assignment is based on the value of instantaneous SNR for each CR user. The estimation of instantaneous value of SNR is thoroughly described in [8]. Here, the estimated SNR values of all users \((\gamma_1,\gamma_2,\ldots,\gamma_N)\) are arranged in an ascending order \((\gamma^1 < \gamma^2 \ldots < \gamma^N)\) and \( N \) cooperative weights (from 1 to \( N \)) are allocated for all users. Whereas; for the user with the lowest \( \gamma_j \) (\( \gamma^1 \)) we assign the weight 1 and the weight \( N \) is allocated for the CR user with the highest SNR (\( \gamma^N \)). The FC’s metric for global decision about the channel status is the weighted summation of the energies and can be written as

\[
Y_w = \sum_{i=1}^{N} k E^i \quad ; \quad (\gamma^1 < \gamma^2 < \gamma^3 \ldots < \gamma^N)
\]

where \( E^i \) s are the arranged version of \( E_j \) s in an ascending order of \( \gamma_j \) s. More precisely, \( E^1 \) is the received energy from the user with the lowest SNR and \( E^N \) is the received energy from the user with the maximum SNR.

Due to the fact that all measured energy values are independent random variables following Gaussian distribution, thus, the metric \( Y_w \) should also follow a Gaussian distribution as

\[
Y_w \sim \begin{cases} 
N(m_o, \sigma_o^2) & H_0 \ \\
N(m_j, \sigma_j^2) & H_1 
\end{cases}
\]

where

\[
m_o = \mu_o \sum_{j=1}^{N} j = \mu_o \frac{N(N+1)}{2} \\
\sigma_o^2 = \sigma_0^2 \sum_{j=1}^{N} j^2 = \sigma_0^2 \frac{N(N+1)(2N+1)}{6} \\
m_j = \mu_j \sum_{j=1}^{N} j = \mu_j \frac{N(N+1)}{2} \\
\sigma_j^2 = \sigma_j^2 \sum_{j=1}^{N} j^2 = \sigma_j^2 \frac{N(N+1)(2N+1)}{6}
\]

Let \( \lambda_w \) denote the global threshold to determine channel status by the FC, if \( Y_w > \lambda_w \) then the FC determines that the channel is occupied.

B) Deriving an Optimum Threshold

The total error probability can be written as

\[
Q_e = Q_{fa}P(H_0) + Q_{m}P(H_1) \\
= P(Y_w > \lambda_w | H_0)P(H_0) \\
+ P(Y_w < \lambda_w | H_1)P(H_1)
\]

\[
= \int_{\lambda_w}^{\infty} \frac{1}{\sqrt{2\pi} \sigma_o^2} \exp\left(-\frac{(x-m_o)^2}{2\sigma_o^2}\right)dx \\
+ \int_{-\infty}^{\lambda_w} \frac{1}{\sqrt{2\pi} \sigma_j^2} \exp\left(-\frac{(x-m_j)^2}{2\sigma_j^2}\right)dx
\]

The optimum threshold \( \lambda_w^* \), to achieve the minimum total error probability \( Q_e \) is obtained as follows

\[
\frac{\partial Q_e}{\partial \lambda_w} = 0 \Rightarrow \lambda_w^* = \frac{m_o \sigma_o^2 - m_j \sigma_j^2 + \psi}{\sigma_o^2 - \sigma_j^2}
\]

where

\[
\psi = \left[ m_o \sigma_o^2 - m_j \sigma_j^2 \right]^2 \left( \sigma_j^2 - m_j \sigma_j^2 \right) \left( \sigma_j^2 - m_j \sigma_j^2 \right) \ln \left( \frac{\sigma_j^2 P(H_0)}{\sigma_o^2 P(H_1)} \right)
\]

IV. SIMULATION RESULTS AND DISCUSSIONS

The numerical simulation is provided to illustrate the usefulness of the proposed method. The existence of an optimum threshold that minimizes total error probability is also investigated. Gaussian noise samples with zero mean and unit variance are assumed and the channel between PU and all CR users is considered as a Rayleigh fading channel. The total number of cooperative CR users is fixed in \( N = 10 \) and the number of samples within a detection interval (\( M \)) is equal to 12. The prior probabilities of sensing channel \( P(H_0) \) and \( P(H_1) \) are assumed to be 0.8 and 0.2,
respectively. All parameters are constant unless otherwise specified.

As depicted in Fig. 3, we compare the performance of the proposed weight assignment method with conventional EGC on the global detection probability versus the average SNR. The average SNR is varying from -10 to 5 dB (here, the threshold is not optimum and obtained from constant false alarm rate for $Q_{fa} = 0.1, 0.3$. The obtained results indicate that, the detection improvement gained by the proposed weighted method is considerably remarkable especially in low SNR region.

Fig. 4 displays the Receiver Operating Characteristic (ROC) for conventional EGC and the proposed methods for two average SNRs -10dB and -5dB. As shown in the figure, for a given value of total false alarm probability $Q_{fa}$ the global detection rate of the proposed method is improved.

Fig. 5 shows the global false alarm, miss detection, and error probabilities versus the decision threshold environment with the average SNR, -5dB. The global false alarm rate is decreased while, the miss detection rate is increased with increasing the threshold value. Obviously, for a smaller value of threshold, the error probability is 0.8 (corresponds to $P(H_0)$) and with increasing the threshold value, the error probability reaches 0.2 ($P(H_1)$). In addition, the shape of $Q_e$ is convex and minimum value of the error is obtained at the optimum threshold point.

The total error probability versus average SNRs for conventional EGC and the proposed method with optimum threshold is shown in Fig. 6. As shown in the figure, the proposed method significantly improves the performance of cooperative sensing.
V. CONCLUSIONS

In this study, Cooperative Spectrum Sensing (CSS) over Rayleigh fading channel was investigated and simulated in a centralized Cognitive Radio (CR) network. As the main contribution, a new weighted soft-decision combining scheme for infrastructure-based CR network was suggested and an optimum threshold to minimize the total error probability is provided. In the proposed method, all of the CR users are ranked based on their instantaneous SNRs and the allocated ranks were used as the dynamic weights in CSS process. The mathematical expression for the new weight model and global error probability was also investigated. Finally, it was concluded that the proposed weighted soft-decision approach improves the cooperative sensing performance compared with conventional equal gain combining scheme.

REFERENCES