ISSN 2047-3338

Sensor Node Localization based on Local Observations using Finite Element Method in Wireless Sensor Network

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***Abstract*— Location information of sensor node is necessary in wireless sensor network since data transmission between nodes may be senseless without knowing the accurate locations of the nodes in network keeping in the view of various applications of WSN such as military surveillance, localization, tracking, battlefield monitoring, structural health monitoring, routing etc. In this paper, we propose to investigate the physical field model for location estimation, i.e. localization, in the form of partial differential equation (PDE). Within the proposed framework, the estimation algorithm of sensor node localization problem using finite element method (FEM) has been employed to obtain the solution based on physical phenomena (e.g., temperature) governed by discretizing the 1–D heat equation. The computational results illustrates that the significant effect and better accuracy using elegant finite element approach by means of simulation results.**

***Index Terms*— WSN, Localization, Heat Equation and FEM**

I. INTRODUCTION

A fast growing field which consists of sensing, computing, communication, actuation and power components, is known as Wireless Sensor Network (WSN) and the respective nodes are sensor nodes. With the low-power circuit and network technologies, tens to thousands of such nodes, in WSN, communicate through wireless channels for information sharing and cooperative processing. Sensor nodes form a sensor network equipped with a processor, memory, wireless communication capabilities, sensing capabilities and a power source (battery) on board. In environmental sensing applications such as bush fire surveillance, water quality monitoring and precision agriculture, for example, sensing data without knowing the sensor location is meaningless.

Localization refers to the process of estimating the locations of sensors using measurements between neighbouring sensors such as distance measurements and bearing measurements [9]. Localization methods could be classified in terms of *active localization* and *passive localization*. The active localization methods estimate the locations based on signals that are artificially stimulated and measured by the sensor network, e.g., artificially generated acoustic events. That means, the localization is performed in controlled environments and incurs significant installation and maintenance costs. The passive localization methods occur in a non-controlled environment, where stimuli are generated in a natural and autonomous fashion. The advantage of passive methods is that they do not need additional infrastructure, and thus keep the installation and maintenance costs at a very low level [3]. In addition, location estimation may enable applications such as inventory management, intrusion detection, road traffic monitoring, health monitoring, etc. Nodes may be either static, in most existing senor network, or mobile are deployed on cellular phones.

Keeping in the view of passive localization of sensor nodes, we present model-based approach based on local observations in terms of physical phenomena (e.g., temperature). Mathematical and computational modeling is an art which translate the real life facts into the mathematical problems, solving the mathematical problems and interpret the result in terms of real world problems for the better understanding of society. The problems of computer science and engineering pose the new challenges for mathematical and computational models. Wireless Sensor Network (WSN) is an emerging interdisciplinary field which involves mathematical modeling of localization especially generates a physical field model. The problem of field estimation is similar to the problem of solving a partial differential equation with initial and boundary conditions. The recent advent of Wireless Sensor Networks (WSNs) offers a significantly different yet attractive approach to field estimation [18].

Finite element method (FEM) is one of the most flexible and attractive approach for solving localization problem in the form of partial differential equation. The finite element method is an advanced mathematical cum numerical technique for solving boundary value problems [1], [2], [20]. It uses variational methods (i.e., the Calculus of variation) to minimize an error function and produce a stable solution. The method involves dividing the solution domain into a finite number of simple sub domains known as elements and to work an appropriate method of [20] constituting solution in each sub region or group or sub regions. The approximation error to be orthogonal to this subspace, the FEM reduces the boundary value problem to a square system of linear equations.

Mathematics offers many significant and striking techniques for field estimation in Wireless sensor network's (WSNs). The work has focused on the creation of an information field useful to mobile agents, human or machines, which accomplish tasks based on the information, provided by the sensor network [4]-[6], [12]-[17] and [21], [22]. In order to address sensor networks in a comprehensive manner, the sensor network community has initiated a research program that includes work in the areas of sensor network architectures, programming systems, reference implementations, hardware.

In this paper, we propose a novel framework for field estimation based on the combination of WSN field measurements with a physical model in the form of a partial differential equation. The mathematical formulation of the temperature problem as a constrained optimization problem, in which the constraints originate from a finite element model of the partial differential equation subject to initial and boundary conditions. The objective is to minimize the error between the estimated and actual/true values at the sensor nodes in the form of temperature. The proposed framework is derived here for one particular type of PDE, namely the one-dimensional (1-D) heat equation, to generate the special information of WSN field measurements [3, 7]. The finite element method has been employed to obtain the solution of one dimensional problem of temperature between sensor nodes. A computer program has been developed in MATLAB 7.11 for the problem and simulated on Core i3 processor with 2.13 GHz processing speed, 64-bit machine with 320 GB memory.

II. PROBLEM FORMULATION

Given a strong model of the physical phenomenon and a set of sensor nodes on unknown, but fixed, locations and use this computational model to determine the sensor node locations [8, 10]. Let us consider 1-D heat equation in circular region is given by [1]-[3]:

$$\frac{\partial T}{\partial t} = D \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r} \right) + s(r,t) \qquad (1)$$

where, D is the thermal conductivity. The temperature $T(x,t)$ is measured at J discrete locations (x_j, y_j) $j = 1, 2.....J$. Each of the J sensor nodes provides N field measurements (with $1N \times 1 = [1...1]T$), $s(r,t)$ is the source term. It is assumed that, at time t = 0 ms, the cell maintains initial temperature of 0.1. Thus the initial condition along the time is taken as [3], [11]:

$$T_{t=0} = 0 \qquad (2)$$

The first boundary of temperature between sensor nodes is assumed at $(r \to 0)$. The boundary condition is taken as [3], [18]:

$$\lim_{r \to 0} \left(-D \frac{\partial T}{\partial n} \right) = 40°c \qquad (3)$$

where, n is normal to the surface.

At another boundary, it is assumed to remain at background temperature of T_∞ i.e. 0 degree c. Thus we have:

$$\lim_{r \to 1} T = 0°c \qquad (4)$$

A. Discretization of the region

The solution region is divided into 10 linear elements, as shown in Figure 1. The numbers inside the circles denote the element number [3], [7].

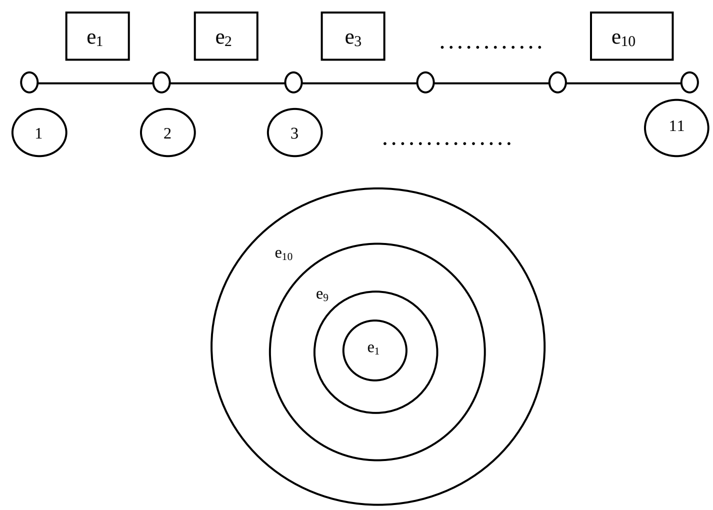


Figure 1: Discretization of the solution region

The discretized variational form of equation (1) can be written as:

$$I^{(e)} = \frac{1}{2} \int_L \left\{ \left(r \frac{\partial T^{(e)}}{\partial r} \right)^2 + \frac{1}{D} T^{(e)} \frac{\partial T^{(e)}}{\partial t} \right\} dx - s(r,t) \mu^{(e)} T^{(e)} \qquad (5)$$

where, e = 1, 2, ..., 10. In the term outside the integral, $\mu^{(e)} = 1$ for e = 1 and $\mu^{(e)} = 0$ for rest of the elements. The shape function of concentration variation within each element is defined by [1]-[2], [20]:

$$T^{(e)} = c_1^{(e)} + c_2^{(e)} r \qquad (6)$$

$$T^{(e)} = P^T c^{(e)} \qquad (7)$$

where, $P^T = \begin{bmatrix} 1 & r \end{bmatrix}$ and $c^{(e)^T} = \begin{bmatrix} c_1^{(e)} & c_2^{(e)} \end{bmatrix}$ (8)

From equations (6) and (7), we get

$$\overline{T^{(e)}} = P^{(e)} c^{(e)} \qquad (9)$$

where, $\overline{T^{(e)}}=\begin{bmatrix}T_i\\T_j\end{bmatrix}$ and $P^{(e)}=\begin{bmatrix}1 & r_i\\1 & r_j\end{bmatrix}$

From the equation (7), we have

$$c^{(e)}=R^{(e)}\overline{T^{(e)}} \tag{10}$$

where, $R^{(e)}=P^{(e)^{-1}}$ (11)

Substituting $c^{(e)}$ from equation (6), (8) and (9) we get

$$u^{(e)}=P^T R^{(e)}\overline{T^{(e)}} \tag{12}$$

Now the integral $I^{(e)}$ can be written in the form:

$$I^{(e)}=I_k^{(e)}+I_\lambda^{(e)}-I_z^{(e)} \tag{13}$$

where,

$$I_k^{(e)}=\frac{1}{2}\int_{r_i}^{r_j}\left\{\left(r\frac{\partial T^{(e)}}{\partial r}\right)^2\right\}dr \tag{14}$$

$$I_\lambda^{(e)}=\frac{1}{2}\int_{r_i}^{r_j}\left[T^{(e)2}\frac{\partial T^{(e)}}{\partial t}\right]dr \tag{15}$$

$$I_z^{(e)}=\frac{1}{2}\mu^{(e)}\frac{s(r,t)T_0}{D}T^{(e)}\Big|_{r=1} \tag{16}$$

Now, we extremize I w.r.t. each nodal temperature T_i, as given below:

$$\frac{dI}{dT}=\sum_{e=1}^{N}\overline{M}^{(e)}\frac{dI^{(e)}}{d\overline{T}^{(e)}}\overline{M}^{(e)T}=0 \tag{17}$$

where, $\overline{M}^{(e)}=\begin{bmatrix}0 & 0\\1 & 0\\. & .\\0 & 1\\0 & 0\end{bmatrix}$ *and* $I=\sum_{e=1}^{11}I^{(e)}$ $\bar{T}=\begin{bmatrix}T_1\\T_2\\.\\.\\.\\T_{11}\end{bmatrix}$

$$\frac{dI^{(e)}}{d\overline{T}^{(e)}}=\frac{dI_k^{(e)}}{d\overline{T}^{(e)}}+\frac{dI_\lambda^{(e)}}{d\overline{T}^{(e)}}-\frac{dI_z^{(e)}}{d\overline{T}^{(e)}} \tag{18}$$

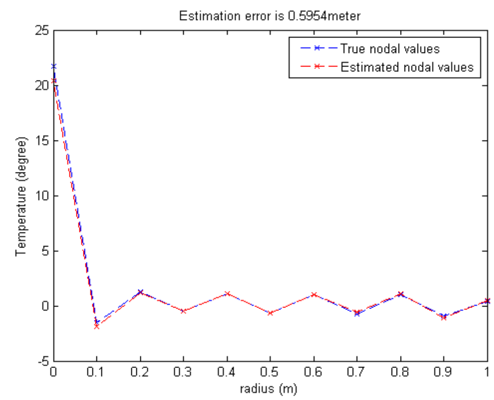
This leads to a following system of linear differential equations [20].

$$\left[\overline{X}\right]_{11\times11}\left[\overline{T}\right]_{11\times1}+\left[\overline{Y}\right]_{11\times11}\left[\frac{\partial\overline{T}}{\partial t}\right]_{11\times1}=\left[\overline{Z}\right]_{11\times1} \tag{19}$$

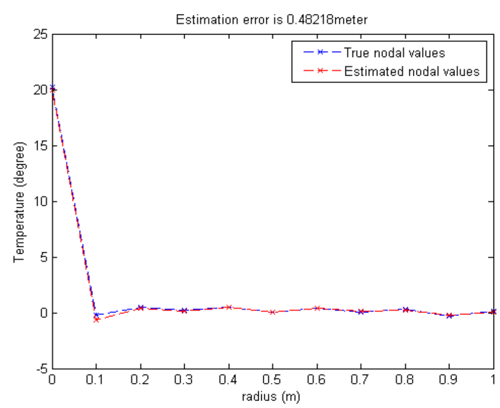
where, $\overline{T}=T_1T_2.............T_{11}$, $\overline{X}$ and $\overline{Y}$ are the system matrices, and $\overline{Z}$ is system vector. The Crank Nicolson Method has been used along the time to obtain the solution of system (19).

III. RESULTS AND DISCUSSION

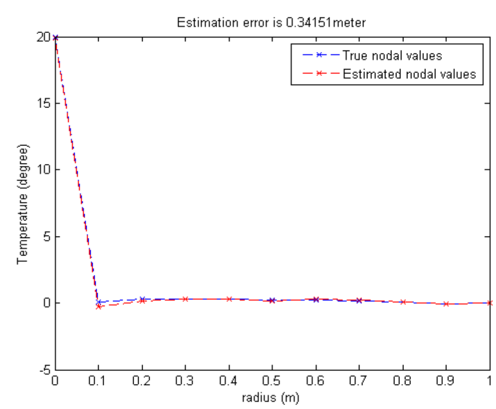
In practical implementation, the radius of circle and time is taken 1 meter (m) and 10 milliseconds (ms), respectively [3], [18]. The nodes are spread in the space, uniformly in circular region. The noisy input function is given by $s(r,t)=\exp(-t)$.



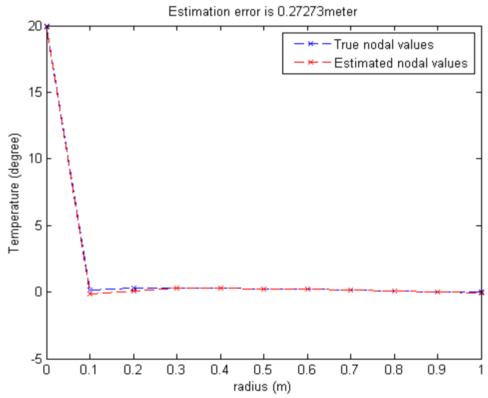
2(a)



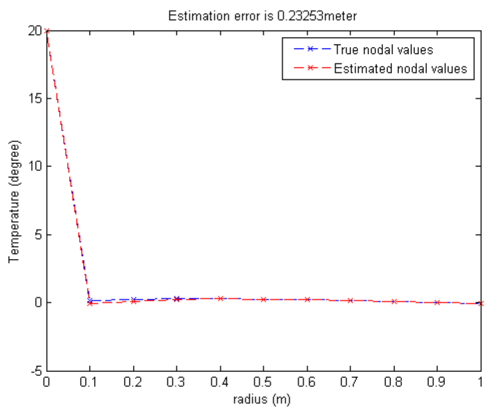
2(b)



2(c)



2(d)

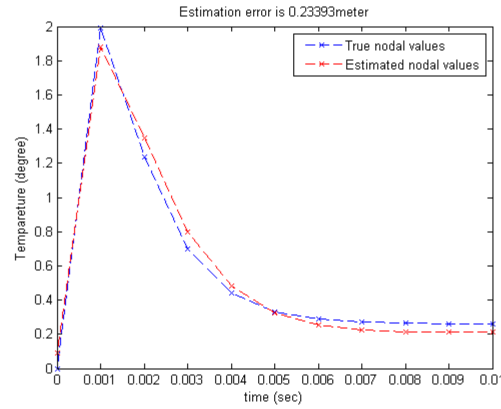


2(e)

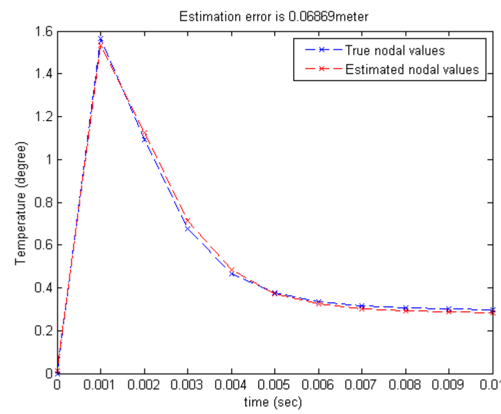
Figure 2: Radial distribution of temperature for different values of time (in milliseconds): (a) t = 2ms, (b) t = 4ms, (c) t = 6ms, (d) t = 8ms and (e) t = 10 ms

Figure 2(a), 2(b), 2(c), 2(d) and 2(e) shows radial and temperature distribution between the sensor nodes for different values of time t = 2ms, 4ms, 6ms, 8ms and 10ms, respectively. These figures illustrate the error analysis between true and predicted nodal values in terms of temperature and it is found that the error estimation is higher, for lower value of time t = 2ms, is 0.5954m and lower, for higher value of time t = 10 ms, is 0.2353m. Therefore, error estimation is inversely proportional to the time. On the other hand; figure 3(a), 3(b), 3(c), 3(d) and 3(e) shows temporal and temperature distribution between the sensor nodes for different values of radius r = 0.2m, 0.4m, 0.6m, 0.8m and 1m, respectively.

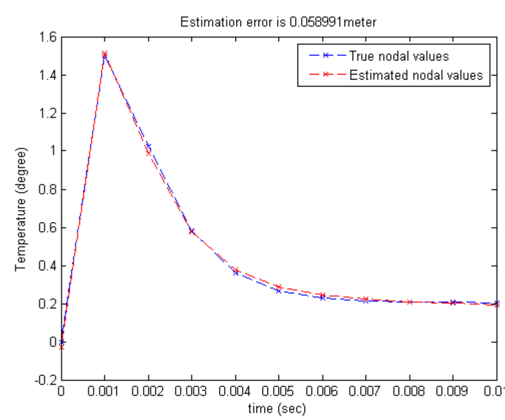
The estimation results i.e. error analysis between true and predicted points are visualized in Figure 3 and it is found that the solution, in terms of temperature distribution between the sensor nodes, is estimated even at actual points with an appropriate certainty between r = 0.2m to r = 0.6m and after that it also gives some uncertainty from r = 0.8m to r = 1m. Furthermore, it is clear that the measurements of the estimated solution between the sensor nodes can be significantly affected.



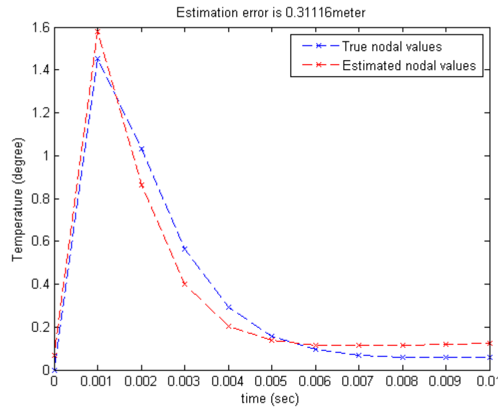
3(a)



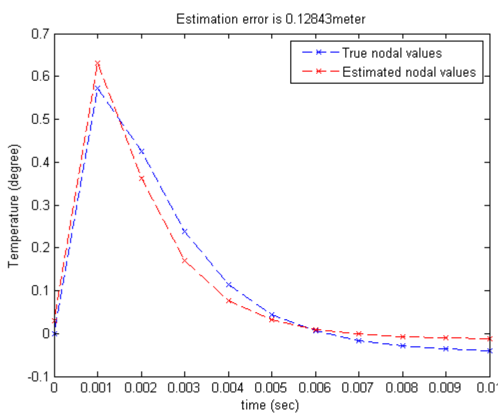
3(b)



3(c)



3(d)



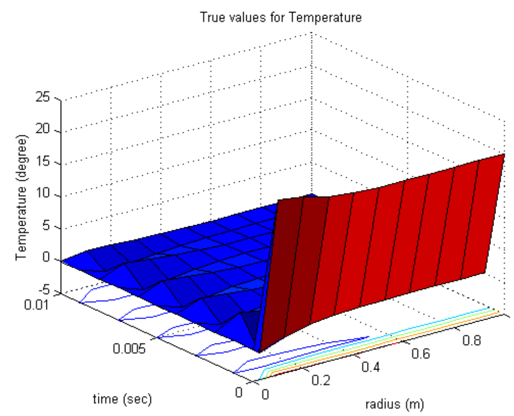
3(e)

Figure 3: Temporal distribution of temperature for different values of radius (in meters): (a) r = 0.2m (b) r = 0.4m (c) r = 0.6m (d) r = 0.8m and (e) r = 1m

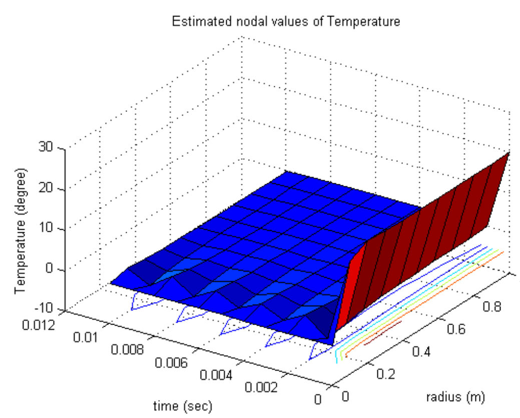
The three dimensional visualization of radial and temporal distribution for temperature distribution between true and estimated nodal points, is depicted in Figure 4. Figure 4(a) represents for temperature distribution at true nodal values and Figure 4(b) represents for temperature distribution at estimated nodal values. It improves the solution of temperature distribution for node localization problem of wireless sensor networks.

To evaluate the performance of the proposed estimation method, sensors nodes are randomly deployed in a finite space and heat equation obtains predicted temperatures for sensor nodes at given assumed locations then uses a distance norm to obtain an estimation between the actual and predicted temperature values. Finally, the objective is to determine the minimum error. If the best estimation is within a guesses is certain threshold then the algorithm returns the locations that best fit to the actual data. Otherwise, new guesses are generated from perturbations of the best guesses and again generates the new random samples with increasing values.

By comparing the above estimations, it is found that by increasing the values of random selections of locations, the computational error between true nodes and predicted nodes becomes minimize. It means that error accuracy depends on best selection of random locations i.e. the error deduction also depends on the selection of data in large amount of sensor locations.



4(a)



4(b)

Figure 4: Radial and temporal distribution of temperature: (a) True nodal points and (b) Estimated nodal points

IV. CONCLUSION AND FUTURE WORK

In this paper, we introduced a new framework for sensor node localization in WSN using FEM. For simulations we consider 1–D field, with initial boundary values, in the circular region governed by a heat equation and it illustrates that the proposed FEM-constrained estimation algorithm consistently outperforms than the estimation method based on WSN measurements only and under certain conditions. Furthermore, we can propose model needs to be generalized in the case of dynamic fields governed by heat equation that also include time derivatives in two and three dimensional problems. We can also focus to the geometry of the region and different parameters related to sensor networks could also be included.

ACKNOWLEDGMENT

I am highly thankful to the faculty members of the Department of Computer Science & Engineering, S. R. Group of Institutions, Jhansi (UP) for their valuable support.

REFERENCES

[1] A. Tripathi and N. Adlakha, *Finite Element Model to Study Calcium Diffusion in Neuron Cell in Presence of Excess Buffer for One Dimensional Steady State* Global Journal of Computational Science & Mathematics (GJCSM), I(1), 21-30, 2011.

[2] B. K. Jha, N. Adlakha, and M. N. Mehta, *Finite element model to study calcium diffusion in astrocytes* Int. J. of Pure and Appl. Math.78(7), 945-955, 2012.

[3] F. Sawo, K. Roberts, and U. D. Hanebeck, Bayesian estimation of distributed phenomena using discretized representations of partial differential equations, in *Proc. 3rd Int. Conf. Informat. Control Autom. Robot. (ICINCO '06)*, Setubal, Portugal, 2006, 16–23.

[4] G.J. Barlow, T.C. Henderson, A.L. Nelson, and E. Grant Dynamic leadership protocol for s-nets. In *IEEE Conference on Robotics and Automation*, New Orleans, LA, April 2004.

[5] H. C. Elman, D. J. Silvester, and A. J. Wathen, *Finite elements and fast iterative solvers: with applications in incompressible fluid dynamics*. Oxford: Oxford University Press, 2005.

[6] H. Zhang, J. M. F. Moura, and B. Krogh, Dynamic field estimation using wireless sensor networks: Tradeoffs between estimation error and communication cost, *IEEE Trans. Signal Process.*, 57(6), 2383–2395, 2009.

[7] I. Dokmani´c, J. Ranieri, A. Chebira and M. Vetterli Sensor Networks for Diffusion Fields: Detection of Sources in Space and Time: Forty-Ninth Annual Allerton Conference Allerton House, UIUC, Illinois, USA September 28 - 30, 2011

[8] J. Ranieri, A. Chebira, Y. M. Lu, and M. Vetterli, Sampling and reconstructing diffusion fields with localized sources, in *Proc. 2011 IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP '11)*, Prague, Czech Republic, 2011, 4016–4019.

[9] N. Patwari, A. O. Hero, M. Perkins, N. S. Correal, and R. J. O'Dea Relative location estimation in wireless sensor networks, IEEE Transactions on Signal Processing, 51, 2137-2148, 2003.

[10] S. Aldalahmeh, M. Ghogho, and A. Swami, Fast distributed detection, localization, and estimation of a diffusive target in wireless sensor networks, in *Proc. 7th Int. Symp. Wireless Commun. Syst. (ISWCS '10)*

[11] S. C. Brenner and L. R. Scott, The mathematical theory of finite element methods. New York: Springer, 2008.

[12] T.C. Henderson and E. Grant. Gradient calculation in sensor networks. In *IEEE Conference on Intelligent Robots and Systems*, Sendai, Japan, 2004.

[13] T.C. Henderson, E. Grant, K. Luthy, and J. Cintron. Snow monitoring with sensor networks. In *IEEE Workshop on on Urban Search and Rescue Performance Measures for Intelligent Systems*, February 2006.

[14] T.C. Henderson, R. Venkataraman, and G. Choikim. Reaction diffusion patterns in smart sensor networks. In *IEEE Conference on Robotics and Automation*, New Orleans, LA, 2004.

[15] Thomas C. Henderson. Leadership protocol for s-nets. In *Proc Multisensor Fusion and Integration*, 289-292, Baden-Baden, Germany, 2001.

[16] Thomas C. Henderson, Mohamed Dekhil, Scott Morris, Yu Chen, and William B. Thompson. Smart sensor snow *IEEE Conference on Intelligent Robots and Intelligent Systems*, 1998.

[17] Thomas C. Henderson, Jong-Chun Park, Nate Smith, and Richard Wright. From motes to java stamps: Smart sensor network testbeds. In *Proc of IROS*, Las Vegas, NV, IEEE, 2003.

[18] T. V. Waterschoot and G. Leus, "Static field estimation using a wireless sensor network based on the finite element method", in *Proc. Int. Workshop Comput. Adv. Multi-Sensor Adaptive Process. (CAMSAP '11)*, San Juan, PR, USA, Dec. 2011

[19] T. Zhao and A. Nehorai, Distributed sequential Bayesian estimation of a diffusive source in wireless sensor networks, *IEEE Trans. Signal Process*, 55(4), 1511–1524, 2007.

[20] S. S. Rao The Finite Element Method in Engineering Elsevier Science and Technology Books, 2004

[21] Y. Chen and Thomas C. Henderson. S-nets: Smart sensor networks. In *Proc International Symp on Experimental Robotics*, pages 85.94, Hawaii, Dec 2000.

[22] Yu Chen. Snets: Smart sensor networks. Master's thesis, University of Utah, Salt Lake City, Utah, December 2000.

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