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Performance Evaluation of Biorthogonal Filters using Adaptive Synthesis Filter Banks for Image Compression

Abdul Nadeem[1] and Yangyu Fan[2]
[1,2]School of Electronics and Information, Northwestren Polytechnical University, Xi'an, Shaanxi 710072, China
[1]labraznad@yahoo.com, [2]fan_yangyu@sina.com

***Abstract*– This paper evaluates the performance of various biorthogonal filters using adaptive synthesis filter banks for image compression. The adaptive synthesis filter banks are used to exploit the phase diversity of synthesis filters in subband coding system. Odd length biorthogonal filters are used because even length filters exhibit fractional delay in compression. It is shown that adaptive synthesis filters derived from higher order biorthogonal filters with higher degree of smoothness display significant objective improvements over lower order biorthogonal filters in image compression.**

***Index Terms*– Biorthogonal Filters, Smoothness, Adaptive Synthesis Filter Banks, Objective Performance and Reconstruction Error**

I. Introduction

WAVELET filter banks have gained significant attention over past two decades due to their suitability for a number of important signal and image processing tasks including image coding. The principle behind the wavelet transform [1-3] is to hierarchically decompose an input signal into a series of successively lower resolution reference signals and their associated detail signals. At each level, the reference signal and detail signal (or signals in the separable multidimensional case) contain the information needed to reconstruct the reference signal at the next higher resolution level. Efficient image coding is enabled by allocating bandwidth according to the relative importance of information in the reference and detail signals.

Wavelet filters are designed using associated continuous scaling functions and iterations. The filters in filter banks do not have to be associated with a single filter or basis function. They can be designed and optimized in many ways. However, the most commonly used image compression systems employ filters with perfect reconstruction (PR), finite impulse response (FIR), and linear phase, and they are nonunitary (biorthogonal). Important properties of wavelet functions in image compression applications are compact support (lead to efficient implementation), symmetry (useful in avoiding dephasing in image processing), orthogonality (allow fast algorithm), regularity, and degree of smoothness (related to filter order or filter length) [4].

Orthogonal wavelets are compactly supported and correspond to finite impulse response (FIR) filters. A major disadvantage of these wavelets (except Harr wavelet) is their asymmetry, which can cause artifacts at borders of the wavelet subbands. Symmetry in wavelets can be obtained only if either compact support or orthogonality is compromised. Compact support and symmetric wavelets are generally desired for image compression. Hence, relaxing the orthogonality condition will result in the formation of biorthogonal wavelets [5]. Biorthogonal wavelets can use filters with similar or dissimilar orders for decomposition (Nd) and reconstruction (Nr). Furthermore, odd length biorthogonal filters are preferred as even length filters exhibit fraction delay in compression.

II. Adaptive Synthesis Filter banks

For lossy image compression, quantization is usually applied to the subband signals, which causes quantization errors to occur in the decoded image. With linear filters, these errors are most visible around sharp edges, especially when images are coded at low bit rates [6]. Adaptive synthesis filter banks (in form time varying filter banks) have the ability to improve the reconstruction quality in particular at lower bit rates, by exploiting the phase diversity of synthesis filters in an adaptive mechanism [7].

In adaptive synthesis filter bank, the analysis section is non adaptive i.e., conventional, while the synthesis section is adaptive as shown in Fig 1, which also eliminates the issue of maintaining dynamic synchronization between the analysis and synthesis filters of adaptive filter banks. The synthesis section consists of "n" filters (where n is an odd integer) and it is composed of (n-1) delay filters along with linear phase filters. The synthesis filters are switched adaptively in the reconstruction process with minimum reconstruction error on pixel by pixel basis, such that in smooth regions, linear phase filters are used. When transition regions or object edges are encountered, appropriate delay filters are employed for image reconstruction.

If "K" is the length of the longer analysis filter, then "2K−4" synthesis filters can be designed by means of the

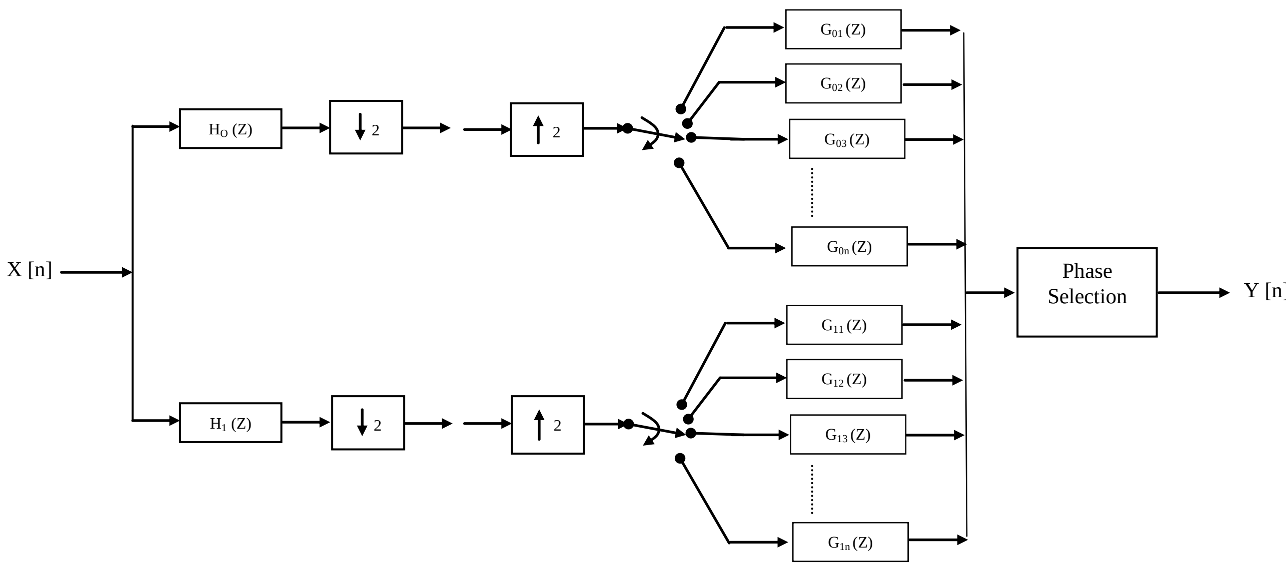

Figure 1. Block diagram conventional analysis and adaptive synthesis filter bank

following time domain equation:

$$\mathbf{A} = \mathbf{SB}$$

where **A** is a block Toeplitz matrix of analysis filter coefficients, **S** is a matrix of synthesis filter coefficients, and **B** is the reconstruction matrix. In a more expanded form, above equation can be expressed as:

$$\underbrace{\begin{bmatrix} \mathbf{P}_0^T & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{P}_1^T & \mathbf{P}_0^T & \vdots & \mathbf{0} & \mathbf{0} \\ \mathbf{P}_2^T & \mathbf{P}_1^T & \vdots & \vdots & \\ \mathbf{P}_3^T & \mathbf{P}_2^T & \ddots & \ddots & \\ \vdots & \vdots & \ddots & \ddots & \\ \mathbf{P}_{K-1}^T & \mathbf{P}_{K-2}^T & \dots & \mathbf{P}_0^T & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{K-1}^T & \ddots & \mathbf{P}_1^T & h_0(0) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{P}_{K-1}^T & h_0(L-1) \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \mathbf{Q}_0 \\ \mathbf{Q}_1 \\ \vdots \\ \mathbf{Q}_{K-2} \\ \mathbf{Q}_{K-1} \end{bmatrix}}_{\mathbf{S}} = \underbrace{\begin{bmatrix} \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{J}_R \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}}_{\mathbf{B}}. \quad (1)$$

The shorter analysis filter is adjusted to "K" by zero padding at the back end. The submatrices of **P** are defined as

$$\mathbf{P}_i^T = [h_0(i) h_1(i)]$$

where $h_0(i)$ and $h_1(i)$ represent the lowpass and highpass analysis filters and the **Q** matrices contain the lowpass and highpass synthesis filter coefficients. The **Q** matrices are given by:

$$\mathbf{Q}_0 = [g_0(0) g_0(1)],$$

$$\mathbf{Q}_1 = [g_1(0) g_1(1)],$$

$$\mathbf{Q}_2 = [g_0(2) g_0(3)],$$

$$\mathbf{Q}_3 = [g_1(2) g_1(3)],$$

and so on until all the synthesis filter coefficients are included. Finally, "$\mathbf{J_R}$" in reconstruction matrix **B** defined as:

$$\mathbf{J}_R = \begin{bmatrix} 0 & 0 & \dots & 1 \\ 0 & & \cdot & 0 \\ \vdots & \cdot & & 0 \\ 1 & 0 & \dots & 0 \end{bmatrix}.$$

The position of "$\mathbf{J_R}$" in the reconstruction matrix **B** controls the phase characteristics of the synthesis filters. Given a desired sample delay, "$\mathbf{J_R}$" is positioned in the (d−1) location of matrix **B,** where "**d**" is the desired system delay.

The synthesis delay filters are computed from common analysis filters by using equation (1) as:

$$\mathbf{S} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{B}.$$

and reconstruction error of these filters is minimized by optimizing their coefficients. For low pass filters, the sum of the odd coefficients and the sum of the even coefficients both are made approximately equal to 0.7071. Similarly, for high pass filters, the sum of the odd coefficients and the sum of the even coefficients are made approximately equal to 0.7071 and -0.7071 respectively.

The synthesis delay filters are then divided into "m/2" groups (where "m" is the number of delay filters and it is even) and each group comprises of two delay filters along with linear phase filters. If "S_d" and "S_p" are delay and linear phase synthesis filters respectively and "d" represents the delay, where

$$\mathbf{d = 1, 2, 3, \ldots\ldots\ldots\ m}$$

then synthesis filters are grouped as

$$\mathbf{G_1 = S_1, S_m, S_p}$$

$$\mathbf{G_2 = S_2, S_{m-1}, S_p}$$

$$\mathbf{G_3 = S_3, S_{m-2}, S_p}$$

$$\vdots \qquad \vdots$$

$$\mathbf{G_{m/2} = S_{m/2}, S_{(m/2)+1}, S_p} \quad \ldots\ldots\ldots\ (2)$$

To improve the reconstruction quality, phase diversity of synthesis delay filters are used in an adaptive mechanism. In the reconstruction process, synthesis filters are selected by using a phase selection criteria i.e. at edges, delay filters are used and in the smooth regions, reconstruction is accomplished by linear phase filters. The selection is based on comparison of three outputs of each sub group on pixel by pixel basis. Let $y_o(i)$, $y_n(i)$ and $y_h(i)$ are the outputs of reconstruction combinations "$\mathbf{S_1}$", "$\mathbf{S_p}$" and "$\mathbf{S_m}$" of the group "$\mathbf{G_1}$" of (2) respectively, then if

$$\mathbf{y_o(i) \approx y_n(i)}$$

and $y_h(i)$ is approximately not equal to $y_n(i)$, then reconstruction is made by "$\mathbf{S_1}$". Similarly, if

$$\mathbf{y_h(i) \approx y_n(i)}$$

and $y_o(i)$ is approximately not equal to $y_n(i)$, then reconstruction is accomplished by "$\mathbf{S_m}$". In case, if none of the delay filters of each sub group meet this criteria, then reconstruction is carried out by linear phase filters "$\mathbf{S_p}$".

Since for "n" synthesis filters, there will be "n^2" possible reconstruction combinations. However, this computational complexity can be reduced to "2n-1" by combining each set of delay filters with corresponding linear phase filters only because other combinations do not significantly contribute in optimal image reconstruction.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \ldots & a_{1m} & a_{1p} \\ a_{21} & a_{22} & a_{23} & a_{24} & \ldots & a_{2m} & a_{2p} \\ a_{31} & a_{32} & a_{33} & a_{34} & \ldots & a_{3m} & a_{3p} \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & a_{m4} & \ldots & a_{mm} & a_{mp} \\ a_{p1} & a_{p2} & a_{p3} & a_{p4} & \ldots & a_{pm} & a_{pp} \end{pmatrix}_{n \times n}$$

Placing of the delay filters is made before the linear phase filters by keeping the delay filters of the groups

$$\mathbf{G_{m/2}, G_{(m/2)-1}, G_{(m/2)-2}, \ldots\ldots\ldots\ G_1}$$

in the order of

$$\mathbf{1^{st}, 2^{nd}, 3^{rd}, \ldots\ldots\ldots\ m/2}$$

positions respectively in the synthesis filter bank and the same placing configuration is implemented up to "level 3" of the subband tree and in all the remaining levels, linear phase filters are used.

III. Simulation results and Conclusions

In this section, we implement adaptive synthesis filter banks derived from various odd length biorthogonal filters. For design examples: we have chosen following biorthogonal filters; "bior 2.2", "bior 2.6", "bior 4.4" and "bior 5.5", whose longer analysis filter lengths "K" are 5, 13, 9 and 11 respectively. So, we can design 6, 22, 14 and 18 different synthesis filters from the respective biorthogonal filters. However, adaptive synthesis filter banks are implemented with a set of four delay synthesis filters alongwith linear phase filters.

Various standard images using the conventional biorthogonal filter banks and adaptive synthesis filter banks are tested and the results are shown in Table 1 and Table 2 respectively for compression ratio 0.5 bits per pixel. The significant improvements in MSE and PSNR using adaptive synthesis filter banks over conventional biorthogonal filter banks are highlighted (in bold) in Table 2.

Adaptive synthesis filter banks derived from higher order biorthogonal filters have shown significant objective improvements especially "bior 5.5". However, overall PSNR of adaptive synthesis filter bank derived from "bior 4.4" is the highest in all the design examples. Higher filter orders give wider functions in the time domain with higher degree of smoothness. Wavelet smoothness also increases with its order and by using longer filters. But complexity of calculating DWT increases by increasing the filter length. Therefore, biorthogonal filters for adaptive synthesis filter banks in image compression applications, we have to find balance between filter length, degree of smoothness, and computational complexity.

Table 1: Results of conventional biorthogonal filter banks for compression ratio = 0.5 bits per pixel

Image	**Bior 2.2**		**Bior 2.6**		**Bior 4.4**		**Bior 5.5**	
	MSE	*PSNR*	*MSE*	*PSNR*	*MSE*	*PSNR*	*MSE*	*PSNR*
Lena	43.2244	31.77	42.8042	31.82	36.1644	32.55	40.3902	32.07
Cameraman	60.7811	30.29	62.9369	30.14	57.6412	32.52	67.2837	29.85
Pepper	48.3596	31.29	49.4214	31.19	43.4659	31.75	50.4251	31.10
Goldhill	41.4043	31.96	41.2782	31.97	37.8357	32.35	44.6201	31.64
House	27.1155	33.80	29.4520	33.44	25.6653	34.04	32.5444	33.01

Table 2: Results of adaptive synthesis filter banks derived from biorthogonal filters for compression ratio = 0.5 bits per pixel

Image	**Bior 2.2**		**Bior 2.6**		**Bior 4.4**		**Bior 5.5**	
	MSE	*PSNR*	*MSE*	*PSNR*	*MSE*	*PSNR*	*MSE*	*PSNR*
Lena	43.2211	31.77	42.7953	31.82	**36.0142**	**32.57**	**39.9667**	**32.11**
Cameraman	60.7786	30.29	62.9160	30.14	**57.2908**	**32.55**	**66.2834**	**29.92**
Pepper	48.3564	31.29	49.4112	31.19	**43.2767**	**31.77**	**49.8996**	**31.15**
Goldhill	41.4028	31.96	41.2759	31.97	**37.7060**	**32.37**	**44.4029**	**31.66**
House	27.1150	33.80	29.4615	33.44	**25.4791**	**34.07**	**31.6373**	**33.13**

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Abdul Nadeem received his B.Sc and M.Sc degrees, both with honors in electrical engineering from University of Engineering and Technology, Taxila in 2000 and 2005 respectively. He is currently pursuing his Ph.D from School of Electronics and Information, Northwestern Polytechnical University. His interests include multirate filter banks, wavelets and their applications in image and video compression.

Yangyu Fan received his Ph.D. from School of Marine Engineering, Northwestern Polytechnical University in December 1999. He is currently a Professor at the School of Electronics and Information of the University and the head of Multimedia & Virtual Reality Laboratory, the Institute of Signal Processing & Wireless Optical Communications. He has published more than 230 papers in various journals and conferences, got 4 invent patents. His research interests include Image Processing, Signal Processing, Virtual Reality, Pattern Recognition and Wireless Optical Communications. He is a member of the Information Processing Branch Association and the DSP Branch Association of China Electronics Society, one of the principals of the Shaanxi Signal Processing Society.