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A Fast Curvelet Transform Image Compression Algorithm using with Modified SPIHT

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***Abstract*– An image compression coding technique involves transforming the image into another domain with Curvelet function and then quantizing the coefficients with modified Set Partitioning in Hierarchical Trees algorithm (SPIHT) has been presented in this paper. Curve functions are effective in representing functions that have discontinuities along straight lines. Normal Wavelet transforms fail to represent such functions effectively. SPIHT has been defined for normal wavelet decomposed images as an embedded quantization process. If the coefficients obtained from Curvelet transform of the image with more discontinuities along straight lines have to subject to quantization process with SPIHT, the existing structure of the SPIHT should be modified to suit with the output of the Fast Curvelet Transform (FCT). In this paper, a modified SPIHT algorithm for FCT coefficients has been proposed. The results obtained from the combination of FCT with modified SPIHT found much better than that obtained from the combination of Wavelet Transform with SPIHT.**

***Index Terms*– C^2-Singularities, FCT, Fast Fourier Transform, SPIHT and Wavelet Transform**

I. INTRODUCTION

THE SPIHT Algorithm [12], [15] as given in the literature is a very useful tool for uniformly quantizing the coefficients obtained from the wavelet sub band decomposition of images. It forms lists using the approximation and N^{th} level decomposition detail coefficients and then checks them for significance against a threshold. Offspring are established using quad tree spatial orientation structures and then each significant coefficient is bit plane coded in the order of descending entropy. Roots are coded prior to the offspring.

The problem in applying such an algorithm to the Curvelet decomposed image is that the form in which curvelet decomposes the image is different from that of wavelets. Also the wavelet decomposition of Radon Projections in the Curvelet analysis is not necessarily dyadic. The approximation & N^{th} level detail coefficients are arranged in the transform matrix in a different order. So LIST formation should be changed. More ever the offspring's are established in a different format.

So modifications must be made in the normal SPIHT Algorithm to make it comply with the Curvelet Transform.

In this paper, we present a new scheme which provides significant improvement in the quality of the compression image in terms of PSNR by Fast Discrete Curvelet transform [13], [14] with SPIHT modified the original algorithm in [15] and modeled the transformed coefficient according to its significance in a 2x2 adjacent offspring group.

In the wavelet transform there is an inability to represent edge discontinuities along the curves. Due to the large or several coefficients are used to reconstruct edges properly along the curves. For this reason, it needs a transform to handle the two dimensional singularities along the sparsely curve. This is the reason behind the birth of Curvelet transform. Here the Curvelet basis elements have wavelet basis and the edge discontinuities and other singularities well than wavelet transform. .

The outline of rest of the paper is organized as follows. Section II discuss the Methodology (theory of Curvelet Transform) section III discuss the SPIHT coding Section IV discuss the Algorithm formulation and modified SPHIT image compression and section V discus the result analysis and comparisons with wavelet transform.

II. METHODOLOGY

A. Image Compression

A common characteristic of most images is that the neighboring pixels are correlated and therefore contain redundant information. The foremost task then is to find less correlated representation of the image. Two fundamental components of compression are redundancy and irrelevancy reduction. Redundancy reduction aims at removing duplication from the signal source (image/video). Irrelevancy reduction omits parts of the signal that will not be noticed by the signal receiver, namely the Human Visual System. In general, three types of redundancy can be identified:

- Spatial Redundancy or correlation between neighboring pixel values.
- Spectral Redundancy or correlation between different color planes or spectral bands.
- Temporal Redundancy or correlation between adjacent frames in a sequence of images (in video applications).

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The Compression techniques are classified as Loss/Lossless Compression & Predictive/Transform Compression [11].

B. Lossless vs. Lossy Compression

In lossless compression schemes, the reconstructed image, after compression, is numerically identical to the original image. However lossless compression can only achieve a modest amount of compression. An image reconstructed following lossy compression contains degradation relative to the original. Often this is because the compression scheme completely discards redundant information. However, lossy schemes are capable of achieving much higher compression. Under normal viewing conditions, no visible loss is perceived (visually lossless).

C. Predictive vs. Transform Coding

In predictive coding, information already sent or available is used to predict future values, and the difference is coded. Since this is done in the image or spatial domain, it is relatively simple to implement and is readily adapted to local image characteristics. Differential Pulse Code Modulation (DPCM) is one particular example of predictive coding. Transform coding, on the other hand, first transforms the image from its spatial domain representation to a different type of representation using some well-known transform and then codes the transformed values (coefficients).

This method provides greater data compression compared to predictive methods, although at the expense of greater computation.

D. Why Wavelets to Curvelets

Many image processing tasks take advantage of sparse representations of image data where most information is packed into a small number of samples. Typically, these representations are achieved via invertible and non- redundant transforms. Currently, the most popular choices for this purpose are the wavelet transform. The success of wavelets is mainly due to the good performance for piecewise smooth functions in one dimension.

The DWT suffers from the following problems:

- *2-D line singularities*- piecewise smooth signals resembling images have 1-Dimentional Singularities. That is, smooth regions are separated by edges, and while edges are discontinuous across, they are typically smooth curves
- *Lack of shift invariance*- these results from the down sampling operation at each level. When the input signal is shifted slightly, the wavelet coefficients amplitude varies largely is explained in [8], [9].
- *Lack of directional selectivity*- as the DWT filters are real and separable the DWT cannot distinguish between the opposing diagonal directions. This was explained in [9].

The first problem of the DWT an anisotropic geometric wavelet transform overcome by the pioneered a new system of representations named ridgelets which deal effectively with line singularities in 2-D using with arbitrary directional selectivity. Those are observed rare cases in real time applications.

In order to analyze local line or curve singularities, consider a partition for the image, and then to apply the ridgelet transform to the obtained sub-images. This block ridgelet based transform, which is named curvelet transform. Apart from the blocking effects; however, the application of this is called first-generation curvelet transform. The second-generation curvelet transform was proposed to handle image boundaries by mirror extension. Previous versions of the transform treated image boundaries by periodization. The second- generation curvelet transform has been shown to be a very efficient tool for many different applications in image processing, seismic data exploration, fluid mechanics, and solving PDEs (partial differential equations). In this survey, we will focus on this successful approach, and show its theoretical and numerical aspects as well as the different applications of curvelets.

The second and third problems can be overcome by the CDWT (Continuous Discrete wavelet transform) which improves the shift invariance & directional selectivity than the separable DWT.

E. Importance of Curvelets over Wavelets

Curvelets will be superior over wavelets in following cases:

i). This transform is optimally sparse representation of objects with edges.

ii). This transform is optimal image reconstruction in severely ill-posed problems.

iii). This transform is optimal sparse representation of wave propagators.

The curvelets offer optimal sparseness for “curve-punctuated smooth” images, where the image is smooth with the exception of discontinuities along C^2 curves. Sparseness is measured by the rate of decay of the m-term approximation (reconstruction of the image using *m* number of coefficients) of the algorithm. Having a sparse representation, along with offering improved compression possibilities and also allows for improving denoising performance as additional sparseness increases the amount of smooth areas in the image. In [6] it was shown that orthogonal systems have optimal m-term approximations that decay in L^2 with rate O (m^{-2}) (as a lower bound). On images with C^2 boundaries, non-optimal systems have the rates:

Fourier Approximation:

$$\left\| f - f_m^F \right\|^2 L^2 \asymp O(m^{\frac{-1}{2}}) \quad (1)$$

Wavelet Approximation:

$$\left\| f - f_m^w \right\|^2 L^2 \asymp O(m^{-1}) \quad (2)$$

Curvelet Approximation:

$$\left\| f - f_m^C \right\|^2 L^2 \asymp O((\log m)^3 (m^{-2})) \quad (3)$$

As seen from the m-term approximations, the Curvelet Transform offers the closest m-term approximation to the lower bound. Therefore, in images with a large number of *C*2 curves (i.e. an image with a great number of long edges), it would be advantageous to use the Curvelet Algorithm.

F. Continuous Curvelet Transform

The Continuous Curvelet Transform has gone through two major revisions. The first Continuous Curvelet Transform [1] (commonly referred to as the "Curvelet '99" transform now) used a complex series of steps involving the ridgelet analysis of the radon transform of an image. Performance was exceedingly slow. The algorithm was modified in 2003 in [3]. The use of the Ridgelet Transform was discarded, thus reducing the amount of redundancy in the transform and increasing the speed considerably. In [8] this new method, an approach of curvelets as tight frames is taken. Using tight frames, an individual curvelet has frequency support in a parabolic-wedge area of the frequency domain.

A sequence of curvelets $\gamma_{j,l,k}$ are tight frames if there exists some value for *A* such that:

$$A \left\| f \right\|^{2}_{L^2} = \sum_{j,l,k} |\langle f, \gamma_{j,l,k} \rangle|^2 \ : \forall f \in L^2 \quad (4)$$

Where each curvelet in the space domain is defined as:

$$\gamma_{j,l,k} = 2^{\frac{2j}{3}} \gamma \left(D_j R_\theta x - k_\delta\right) \quad (5)$$

(With Dj = Parabolic Scaling matrix, R_θ = Rotation matrix, k_δ = translation parameter, γ = the "mother" curvelet Using the property of tight frames, the inverse of the curvelet transform is easily found as:

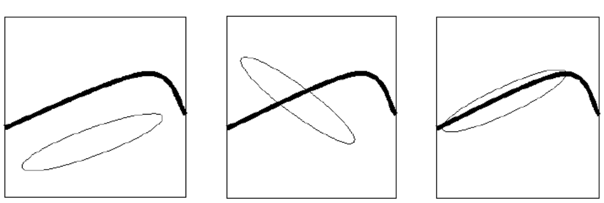
$$f = \sum_{j,l,k} \langle f, \gamma_{j,l,k} \rangle \, \gamma_{j,l,k} \quad (6)$$

In a heuristic argument is made that all curvelets fall into one of three categories.

i). A curvelet whose length-wise support does not intersect a discontinuity. The curvelet coefficient magnitude will be zero. (Fig. 1.a)

ii). A curvelet whose length-wise support intersects with a discontinuity, but not at its critical angle. The curvelet coefficient magnitude will be close to zero. (Fig. 1.b)

iii). A curvelet whose length-wise support intersects with a discontinuity, and is tangent to that discontinuity. The curvelet coefficient magnitude will be much larger than zero. (Fig. 1.c)



(a) (b) (c)

Fig.1. Curvelet Type A, Curvelet Type B and Curvelet Type C

G. Discrete Curvelet Transform

Discrete curvelet transform is implemented using the wrapping based fast discrete curvelet transform. Basically, multi resolution discrete curvelet transform in the spectral domain utilizes the advantages of fast Fourier transform (FFT). During FFT, both the image and the curvelet at a given scale and orientation are transformed into the Fourier domain. The convolution of the curvelet with the image in the spatial domain then becomes their product in the Fourier domain. At the end of this computation process, we obtain a set of curvelet coefficients by applying inverse FFT to the spectral product. This set contains curvelet coefficients in ascending order of the scales and orientations. There is a problem in applying inverse FFT on the obtained frequency spectrum. The frequency response of a curvelet is a trapezoidal wedge which needs to be wrapped into a rectangular support to perform the inverse Fourier transform. The wrapping of this trapezoidal wedge is done by periodically tiling the spectrum inside the wedge and then collecting the rectangular coefficient area in the origin. Through this periodic tiling, the rectangular region collects the wedge's corresponding fragmented portions from the surrounding parallelograms. For this wedge wrapping process, this approach of curvelet transform is known as the 'wrapping based curvelet transform'.

H. Discrete Curvelet Transform Wrapping

Using the theoretical basis in (where the continuous curvelet transform is created), two separate digital (or discrete) curvelet transform (DCT) algorithms is introduced in [4]. The first algorithm is the non equispaced FFT Transform [10], where the curvelet coefficients are found by irregularly sampling the Fourier coefficients of an image. The second algorithm is the Wrapping transform, using a series of translations and a wraparound technique. Both algorithms having the same output, but the Wrapping Algorithm gives both a more intuitive algorithm and faster computation time. Because of this, the Unequispaced FFT [18], [19], [20] method will be ignored in this paper with focus solely on the Wrapping DCT method.

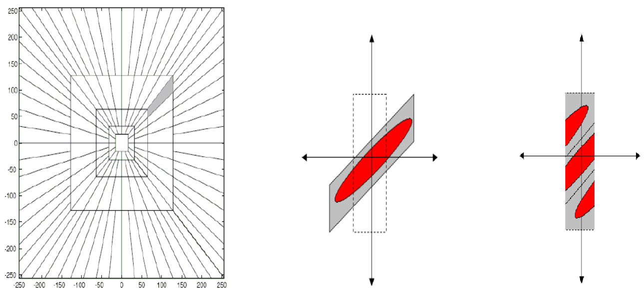
Wrapping DCT Algorithm:

1. Take FFT of the image
2. Divide FFT into collection of Digital Corona Tiles
3. for each corona tile
   (a) Translate the tile to the origin (Fig. 4.)
   (b) Wrap the parallelogram shaped support of the tile around a rectangle centered at the origin
   (c) Take the Inverse FFT of the wrapped support
   (d) Add the curvelet array to the collection of curvelet coefficients.

Inverse Wrapping DCT Algorithm:

1. for each curvelet coefficient array
   (a) Take the FFT of the array.
   (b) Unwrap the rectangular support to the original orientation shape.
   (c) Translate to the original position.

(d) Store the translated array.
2. Add all the translated curvelet arrays.
3. Take the inverse FFT to reconstruct the image



(a) (b) (c)

Fig. 2. (a) Digital Corona of the Frequency Domain, (b) Support of Wedge before Wrapping; and (c) Support of Wedge after Wrapping

I. Fast Discrete Curvelet Transform Operation

Fast Discrete Curvelet transform (FDCT) [5], [6], [7] gives different frequency components locally for analysis and synthesis of digital image in multi-resolution analysis. FDCT is multi-scale geometric transform, which is a multi-scale pyramid with many directions and positions at each length scale. FDCT is basically 2D anisotropic extension to classical wavelet transform that has main direction associated with it. Analogous to wavelet, FDCT can be translated and dilated. The dilation is given by a scale index that controls the frequency content of the curvelet with the indexed position and direction can be changed through a rotation. This rotation is indexed by an angular index. Curvelet satisfy anisotropic scaling relation, which is generally referred as parabolic scaling. This anisotropic scaling relation associated with curvelet is a key ingredient to the proof that curvelet provides sparse representation of the C^2 function away from edges along piecewise smooth curves. FDCT is constructed by a radial window *W* and angular window. The radial window W is expressed as

$$\widetilde{W}_J(w) = \sqrt{(\emptyset_{j+1}^2(w) - \emptyset_j^2(w))}\ ,\ j \geq 0 \quad (7)$$

Where, $\emptyset$ is defined as the product of low-pass one dimensional window.
The angular window V is defined as

$$V_j(w) = V\,(2^{j/2}\ w_2/w_1) \quad (8)$$

Where, W1 and W2 are low pass one dimensional windows. The Cartesian window $\widetilde{U}_{j,l}(w)$ is constructed as

$$\widetilde{U}_{j,l}(w) = W_j(w)\, V_j(S_\theta\, w) \quad (9)$$

Where, S_θ *is* shear matrix, $S_\theta = \begin{bmatrix} 1 & \emptyset \\ \tan\emptyset & 1 \end{bmatrix}$

Shear matrix S_θ is used to maintain the symmetry around the origin and rotation by $\pm\pi/2$ radiance.

The frequency domain definition of digital curvelet is,

$$\varphi_{j,l,k}^{\tilde{D}}[t1,t2] = \widehat{U_{J\,[t1,t2]}}\, e^{-t2\pi[k1t1+k2t2]} \quad (10)$$

Where $\varphi_{j,l,k}$, is a Cartesian window. The Discrete Curvelet transform is expressed as:

$$c^D(j,l,k) = \sum_{0\leq t1,t2<n} f[t1,t2]\, \varphi_{j,l,k}^{\tilde{D}}[t1,t2] \quad (11)$$

Where, $c^D(j,l,k)$ represents curvelet coefficients with *j* is scale parameter, *l* is orientation parameter and *k* is position parameter. f $[t_1, t2]$ is an input of Cartesian arrays. This transform is also invertible. The classical wavelet transform captures the image features only in vertical, horizontal and diagonal directions with isotropic scaling. Wavelets do well for point singularities and not for singularities along curves. Wavelets are not well adapted to edges because of its isotropic scaling. FDCT is applied to a rotated and up sampled high-resolution grid. The high- resolution grid is decomposed at three levels in curvelet domain. In order to interpolate the missing pixels their locations must be determined in each sub band. In curvelet domain missing pixels corresponds to missing coefficients of each sub band. The missing coefficients are interpolated at finest scale. Inverse curvelet transform reconstructs the original high-resolution grid.

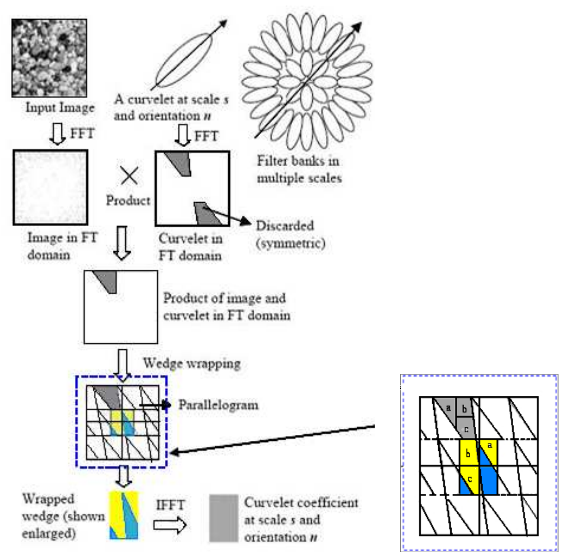


Fig. 3. Image to Curvelet coefficients

III. SPIHT (SET PARTITIONING IN HIERARCHICAL TREES)

SPIHT is an embedded coding technique. In embedded coding algorithms, encoding of the same signal at lower bit rate is embedded at the beginning of the bit stream for the target bit rate. Effectively, bits are ordered in importance. This type of coding is especially useful for progressive transmission using an embedded code; where an encoder can terminate the encoding process at any point. SPIHT algorithm is based on following concepts [15], [16]:

i). Ordered bit plane progressive transmission.
ii). Set partitioning sorting algorithm.
iii). Spatial orientation trees.

SPIHT keeps three lists: List of insignificant pixels (LIP), List of insignificant sets (LIS), and List of significant pixels (LSP).LIP stores insignificant pixels, LSP stores significant pixels and LIS stores insignificant sets. At the beginning, LSP is empty, LIP keeps all coefficients in the lowest sub band, and LIS keeps all tree roots which are at the lowest sub band. The SPIHT algorithm sends the binary representation of the integer value of Curvelet coefficients (bit-plane coder). [17] also present simulations that show the superiority of SPIHT coding over the traditional JPEG During step of initialization, initial value for threshold is determined and initializes with a set containing all the coefficients in lowest subband (LIP). Moreover, initially empty list set in LSP and LIS contains the coordinates of roots of all trees that are of ¾ of lowest subband. In this paper SPIHT is modified with LIP initialization to be inserted in the hybrid coder. The LSP and LIS lists have not been modified, LSP is originally empty due to the approximation subband and offspring of LIS. The approximation subband coefficients values have been not included in order to achieve a better detail subband encoding.

Progressive selection of coefficients such that

$$T_p = \frac{c_{max}}{2^{p+1}} \quad (12)$$

Where p= 0, 1, 2…... P denotes the pass number

$$c_{max} = 2^{\lfloor \log_2 \max \{|c_{i,j}|\} \rfloor} \quad (13)$$

Where $c_{i,j}$ is the coefficient at position (i,j) in the image

$$|c_{i,j}| \geq 2^n, n = n_0, n_0 - 1, n_0 - 2 \quad (14)$$

$$S_n(U_m) = \begin{cases} 1, & \max\{|c_{i,j}|\} \geq T_p, 0 \text{ other wise} \\ & (i,j) \in U \end{cases} \quad (15)$$

Algorithm Steps:

1) select partitions of pixels Um

2) For each n = n_0, n_0-1, n_0-2...
   • If $S_n(U_m)$ = 0 (the set is insignificant) then disregard pixels in Um

   • If $S_n(U_m)$ = 1 (the set is significant) then use recursive algorithm to partition Um

3) Test sets until all significant coefficients found
4) The following sets of coordinates are used to present the new coding method:
   O (i, j): set of coordinates of all offspring of node (i, j)
   D (i, j): set of coordinates of all descendants of node (i, j)
   H (i, j): set of coordinates of all spatial orientation tree roots (nodes in the highest pyramid level)
   L (i, j): D (i, j) – O (i, j) (all descendents except the offspring)
5) Three Lists
   • LIP - list of insignificant pixels
   • LIS - list of tree roots (i, j) of insignificant descendant sets D (i, j) (Type A) or insignificant descendant of offspring sets L (i, j) = D (i, j) - O (i, j) (Type B)
   • LSP - list of significant pixels
6) Lists tested in order LIP, LIS, LSP for efficient embedded coding
7) Initialization of Lists
   • LIP: co-ordinates of all tree roots wavelet example: co-ordinates in coarsest scale subband
   • LIS: co-ordinates of all tree roots with nonempty descendent trees wavelet example: Co-ordinates in coarsest scale subband pointing to descendant trees
   • LSP: empty
8) Sorting Pass
9) Refinement Pass
   • Output n^{th} bit of all LSP members found significant at thresholds greater than 2n
   • Two bit types in stream: significance test bits and refinement bits
10) Quantization Step Update:
   • Decrement the value of n by 1 and go to sorting pass if n is not less than 0

The SPIHT Algorithm is very useful tool for uniformly quantizing the coefficients obtained from the wavelet sub band decomposition of images .It forms lists using the approximation and Nth level decomposition detail coefficients and then checks them for significance against a threshold. Offspring are established using quad tree spatial orientation structures and then each significant coefficient is bit plane coded in the order of descending entropy. Roots are coded prior to the offspring.

The problem in applying such an algorithm to the Curvelet decomposed image is that the form in which curvelet decomposes the image is different from that of wavelets.

i). Also the wavelet decomposition of Radom Projections in the Curvelet analysis is not necessarily dyadic.
ii). The approximation and n^{th} level detail coefficients are arranged in the transform matrix in a different order. So LIST formation should be changed.
iii). And moreover the offspring’s are established in a different format.

IV. ALGORITHM FORMULATION & MODIFIED SPHIT

In this section we described the following coding technique for images with straight singularities along with curves and edges. The proposed is very effective in overcoming the shortcomings of the wavelet transform based coding methods when applied to images with linear Curves. The technique is as follows:

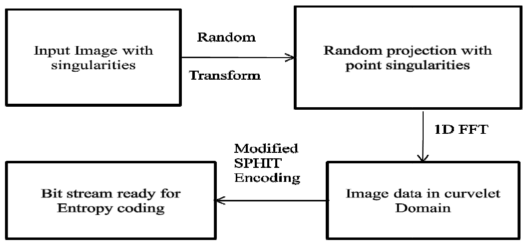


Fig. 4. Proposed Image Compression Techniques

i). Represent the image data as intensity values of pixels in the spatial co-ordinates.
ii). Apply Curvelet Transform on the image matrix and get the Curvelet coefficients of the image.
iii). Quantize the available coefficients using the SPIHT Algorithm, specially modified for the Curvelet Transform.
iv). Use any form of entropy coding on the bit stream available from the SPIHT encoder.

Modified SPHIT: The following are the solutions proposed to address these problems in applying SPIHT to curvelet transformed image:

i). The Curvelet Transform which uses dyadic Decomposition of Radon Projections to the maximum number of levels is chosen for the coding technique.
ii). In Curvelet Transform we find that each column (Wavelet transformed Radon Projection) has one Approximation coefficient, one n^{th} level detail coefficient, two $(n-1)^{th}$ level coefficients, four $(n-2)^{th}$ level coefficients and so on. Hence it can be seen that all approximation coefficients fall in the 1^{st} row of the CT image and all n^{th} level detail coefficients fall in the 2^{nd} row.

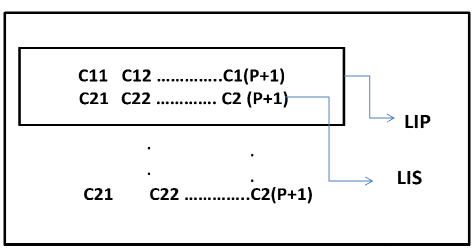


Fig. 5. List Formations in Modified SPIHT Algorithm

So it is very clear that from Fig. 5 that the Modified SPIHT has different type of list contents against Normal SPIHT as described below:

In Normal SPIHT – LIP contains approximation coefficients with n^{th} level detail coefficients. But, LIS will have n^{th} level detail coefficients.

In Modified SPIHT – LIP contains 1^{st} and 2^{nd} row which is again approximation coefficients with n^{th} level detail coefficients. But, LIS will have 2^{nd} row which is n^{th} level detail coefficients.

iii). Every root has its offspring in the same column, which means that the spatial orientation trees are mapped considering each column as a 1-D vector individually. Every n^{th} detail coefficients has two offspring in $(n-1)^{th}$ detail coefficients set lying in the corresponding position from the top, as its root is in the n^{th} level detail coefficients set.

Hence as a generalization offspring can be mapped by the following formula:

$$O(i, j) = \{(2i, j), (2i+1, j)\} \tag{16}$$

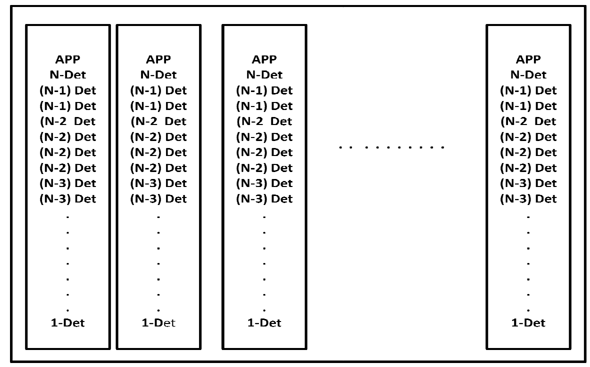


Fig. 6. Image Decomposition and Offspring dependencies using curvelet transform

As a result each parent has two offspring in contrast to the four offspring for each parent in the normal SPIHT encoding. These are the important changes made in the SPIHT procedure to be used for Curvelets Rest of the procedure is identical to the one applied to wavelets including thresholding, refinement and decoding. The changes made in the encoding process must be considered in the decoding process and appropriately reflected in the inverse way for faithful reconstruction.

V. RESULTS AND COMPARISONS

During the process of said compression, the reconstructed image is subject to a wide variety of distortion. Subjective evaluations emphasizes on the visual image quality, which is too inconvenient, time consuming, and complex. The objective image quality metrics like Compression ratio, Peak Signal to Noise Ratio (PSNR), or Mean Squared Error (MSE) are thought to be the best for the image processing application.

Compression ratio: It also known as compression power is a term used to quantify the reduction in image representation size produced by a image compression algorithm .The data compression ratio is analogous to the physical compression ratio used to measure physical compression of substances, and is defined in the same way, as the ratio between the uncompressed image size and the compressed image size:

Compression Ratio = Uncompressed image Size/Compressed image Size

The MSE metric is most widely used for it is simple to calculate, having clear physical interpretation and mathematically convenient. MSE is computed by averaging the squared intensity difference of reconstructed image, $\hat{x}$ and the original image, x. Then from it the MSE is calculated as,

$$MSE = 1/MN[y(i, j) - \hat{y}(i, j)]^2 \qquad (17)$$

where x(i , j) is the original image, $\hat{x}$ (i , j) is the approximated version (which is actually the decompressed image) and M,N are the dimensions of the images Where, MxN is the size of the image and assuming the grey scale image of 8 bits per pixel (bpp), then the PSNR is defined as,

$$PSNR = 10 \log_{10} \left[255^2 / MSE\right] \qquad (18)$$

(a) Clock tower (b) Moon

Fig. 7. Test Images

For testing the performance of the proposed image coding system, the following test images were taken these images in Fig. 7 were subjected to Curvelet transform which uses 2-D dyadic wavelet decomposition. Hence it provided means for SPIHT encoding. The standard parameters like MSE, PSNR and CR were calculated for various numbers of planes excluded during the SPIHT encoding. The results obtained are compared with the SPIHT encoded images after normal 2-D wavelet decomposition up to the equal number of levels. The wavelet named 'db1' was used in both cases.

The results given in Table I clearly show the superiority of Curvelet Transforms over wavelet transforms with straight curves. Moreover the smooth distribution of the intensity levels Contributes to the compaction of most part of the image energy in the low frequency range, so that the compression becomes much easier and effective.

TABLE I
COMPARISON RESULTS OF CLOCK TOWER IMAGE

No. of bit planes excluded	CR		RMSE		PSNR	
	Proposed scheme	Wave let	Proposed scheme	Wave let	Proposed scheme	Wave let
6	49.06	52.08	10.76	19.87	34.15	24.11
5	24.07	26.00	9.29	18.97	34.35	24.57
4	6.63	8.43	7.82	18.62	35.93	25.12
3	1.89	3.69	6.57	17.35	35.14	25.39

Statistics in Table II provides a clear insight in the efficiency of the Curvelet transform in coding the Saturn image which has a curved edge. Even though the boundary is curved, we can very well observe different shades of intensity along straight lines. This aspect has contributed to the success of Curvelet over Wavelets. The tradeoff between Compression ratio and PSNR is very appreciable and encourages the usage of Curvelet transforms for astronomical data like the Moon image.

TABLE II
COMPARISON RESULTS OF MOON IMAGE

No. of bit planes excluded	CR		RMSE		PSNR	
	Proposed scheme	Wave let	Proposed scheme	Wave let	Proposed scheme	Wave let
6	61.80	53.60	12.10	14.68	38.24	29.04
5	57.37	43.89	12.53	14.37	36.98	29.40
4	32.19	20.87	6.49	13.94	34.55	29.11
3	3.64	7.54	6.12	13.60	31.14	26.62

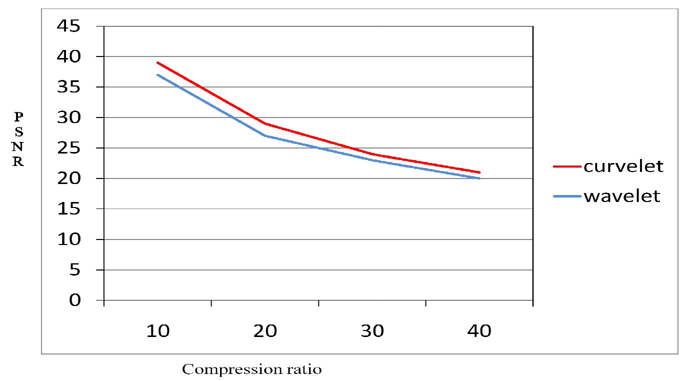


Fig. 8. Comparison of wavelet and proposed algorithm compression ratio with PSNR

VI. CONCLUSIONS

The proposed algorithm of fast Curvelet transform combined with Modified SPIHT scheme gave compression ratios as high as 90:1 with very good PSNR. The results are found to be comparable with conventional wavelet based compression which is more discontinuous with straight line singularity and hence this method further investigation. The Fast curvelet with modified SPHIT is low computational complexity and the computational speed of the algorithm is very good than the wavelet based schemes. Experimental results clearly show that the proposed compression technique results in higher quality reconstructed images compared to that of other algorithms operating at similar bit rates for the class of images where edges are dominant with minimum variation in compression ratio. Thus, we can conclude that FCT with modified SPIHT is to be better compression algorithms in terms of visual and computational performance. From the comparison result the better performance for PSNR is obtained from the Curvelet coder as compared with the Wavelet compression. It is about 10% to 20% improvement in PSNR had arrived from the above algorithm.

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