Realistic Approach of Strange Number System from Unodecimal to Vigesimal

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Abstract—Presently in computer science and technology, number system is based on some traditional number system viz. decimal (base 10), binary (base-2), octal (base-8) and hexadecimal (base 16). The numbers in strange number system (SNS) are those numbers which are other than the numbers of traditional number system. Some of the strange numbers are unary, ternary, ..., Nonary, ..., unodecimal, ..., vigesimal, ..., sexagesimal, etc. The strange numbers are not widely known or widely used as traditional numbers in computer science, but they have charms all their own. Today, the complexity of traditional number system is steadily increasing in computing. Due to this fact, strange number system is investigated for efficiently describing and implementing in digital systems. In computing the study of strange number system (SNS) will useful to all researchers. Their awareness and detailed explanation is necessary for understanding various digital aspects. In this paper we have elaborate the concepts of strange number system (SNS), needs, number representation, arithmetic operations and inter conversion with different bases, represented in tabulated form. This paper will also helpful for knowledge seekers to easy understanding and practicing of number systems as well as to memories them.

Index Terms—Unodecimal, Duodecimal, Tridecimal, Quadrodecimal, Pentadecimal and Vigesimal

I. INTRODUCTION

The concept of number is the most basic and fundamental in the world of science and mathematics, is associated with the concept of a set. A set is meant a collection of definite and separate objects of any kind for which we can decide whether or not a given object belongs. Numeral is the symbol for the idea called number. A number system is the set of symbols used to express quantities as the basis for counting, determining order, comparing amounts, performing calculations, and representing value. It is the set of characters and mathematical rules that are used to represent a number. The first digit in any numbering system is always a zero. The base or radix of a number system is the number of digits present. For example, a base 11 (unodecimal) number contains 11 digits: numeral 0 through 9 and alphabet A, a base 12 (duodecimal) numbers contain 12 digits numeral 0 through 9 and alphabets A and B, a base 13 (tridecimal) number contains 13 digits: numeral 0 through 9 and alphabet A, B and C and so fourth. These are an alphanumeric number system because its uses both alphabets and numerical to represent a particular number.

It is well known that the design of computers begins with the choice of number system, which determines many technical characteristics of computers. But in the modern computer, the traditional number system is only used, especially binary number system. However, research in the field of number systems has continued in modern computer science. A basic motivation of this paper is to overcome a number of essential deficiencies of the binary number system. The most well known of these are:

(a) The sign problem (it is impossible to represent negative numbers and perform arithmetical operations over them in direct code) that complicates arithmetical computer structures.

(b) The problem of zero redundancy (all binary code combinations are permitted).

(c) Limitation of speed of modern computers in performing the arithmetic operations such as addition, subtraction and multiplication suffer from carry propagation delay that does not allow the checking of informational processes in processors and computers effectively.

Due to the increasing the complexity of traditional number system in computing, strange number system is widely used. The first attempt to overcome the sign problem of the binary number system was made a ternary computer, called ‘Setun’ by Nikolai P. Brousentsov in 1958 at Moscow University in Russia. To overcome the problem of zero redundancy, another original discovery in number-system theory, called ‘Tau System’ was made by the American mathematician George Bergman in 1957 and in QSD number system carry propagation chain are eliminated which reduce the computation time substantially and enhancing the speed of the machine [4], [5].

Although many researcher and knowledge seeker know only the traditional number system such as decimal, binary, octal and hexadecimal and are very comfortable with performing operations using this system, it is important for them to understand that traditional number system is not the only
system. By studying other number system such as unodecimal (base-11), duodecimal (base-12), tridecimal (base-13), quadrodecimal (base-14), pentadecimal (base-15), heptadecimal (base-17), octodecimal (base-18), nonadecimal (base-19) and vigesimal (base-20), researcher will gain a better understanding of how number systems work in general. When discussing how a computer stores information, the binary number system becomes very important since this is the system that computers use. It is important that students understand that computers store and transmit data using electrical pulses, and these pulses can take two forms - "on" (1) or "off" (0).

This paper will stimulate the reader’s interest to the strange number system beyond traditional number system. In this particular paper, we are taking under the consideration a tabulated format for few strange number systems. It covers each system’s number representation, their uses, their arithmetic and inter-conversion of numbers from one system to another.

II. NEED OF THE STRANGE NUMBER SYSTEM

The modern digital computer normally deals with the traditional number (i.e., binary, octal, decimal and hexadecimal) as per as computer science and information technology is concern. Apart from these traditional number systems, the strange number system also plays a significant role in computing. The strange number system poses some extra features which distinguish them from the other existing number systems and make them worth an extra look; some of these features include [1]:

- Greater speed of arithmetic operations realization
- Greater density of memorized information
- Better usage of transmission paths
- Decreasing of interconnections complexity and interconnections area
- Decreasing of pin number of integrated circuits and printed boards
- Avoid sign problem and zero redundancy problem

III. CLASSIFICATION OF NUMBER SYSTEM

The basic categorization of number system is two types, i.e., non-positional number system and positional number system.

A. Non-Positional Number System

In a non-positional number system, each number in each position does not have to be positional itself. Every system varies by country and it depends on symbols and values set by the people of that country. In this system, we have symbols such as I for 1, II for 2, III for 3, IIII for 4, IIIII for 5, etc. The unary number system is under this category.

B. Positional Number System

In a positional number system, there are only a few symbols called digits, and these symbols represent different values depending on the position they occupy in the number. The value of each digit in such a number is determined by three considerations:

- The digit itself.
- The position of the digit in the number.
- The base of the number system.

The unodecimal, duodecimal, tridecimal, quadrodecimal, pentadecimal, heptadecimal, nonadecimal and vigesimal number systems are the example of positional number system.

IV. NUMBER REPRESENTATION OF STRANGE NUMBER SYSTEM

In general in a number system with a base or radix n, the digits used are from 0 to n-1 and the number can be represented as: \( x = a_n b^n + a_{n-1} b^{n-1} + \ldots + a_1 b^1 + a_0 b^0 \), where \( x = \text{Number}, b = \text{Base}, a_n, \ldots, a_0 = \text{any digit in that base} \).

Any real number \( x \) can be represented in a positional number system of base "b" by the expression \( x = a_n b^n + a_{n-1} b^{n-1} + \ldots + a_1 b^1 + a_0 b^0 + a_{-1} b^{-1} + \ldots + a_{-(n-1)} b^{-(n-1)} + a_{-n} b^{-n} \).

The point that separates the integer part and fraction part is known as the radix point. The number representation of strange number system is as follows (Table I):

<table>
<thead>
<tr>
<th>Number System</th>
<th>Base</th>
<th>Symbol</th>
<th>Number Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unodecimal</td>
<td>11</td>
<td>0,...,9,A</td>
<td>A×11^2 + 0×11^1 + 1×11^0 + 0×11^-1 + 2×11^-2</td>
</tr>
<tr>
<td>Duodecimal</td>
<td>12</td>
<td>0,...,9,A,B</td>
<td>B×12^2 + 0×12^1 + 1×12^0 + 0×12^-1 + 2×12^-2</td>
</tr>
<tr>
<td>Tridecimal</td>
<td>13</td>
<td>0,...,9,A,...,C</td>
<td>C×13^2 + 0×13^1 + 2×13^0 + 0×13^-1 + 3×13^-2</td>
</tr>
<tr>
<td>Quadrodecimal</td>
<td>14</td>
<td>0,...,9,A,...,D</td>
<td>D×14^2 + 0×14^1 + 3×14^0 + 0×14^-1 + 4×14^-2</td>
</tr>
<tr>
<td>Pentadecimal</td>
<td>15</td>
<td>0,...,9,A,...,E</td>
<td>E×15^2 + 0×15^1 + 4×15^0 + 0×15^-1 + 5×15^-2</td>
</tr>
<tr>
<td>Heptadecimal</td>
<td>17</td>
<td>0,...,9,A,...,G</td>
<td>G×17^2 + 0×17^1 + 5×17^0 + 0×17^-1 + 6×17^-2</td>
</tr>
</tbody>
</table>
The study of strange number systems (SNS) is useful to the researchers of computing due to the fact that different fields have been discussed. In this section, strange number systems and their applications in the area of mathematics, according to the show’s creator. The unodecimal number system is also used to check digit for ISBN. People count by ten’s and machines count by two’s - that pretty much sums up the way we do arithmetic on this planet. The cultural preference for base 10 and the engineering advantages of base 2 have nothing to do with any intrinsic properties of the decimal and binary numbering system. But there are countless other ways to count using strange number system (SNS). These number systems are not as widely known as their decimal and binary cousins, but they have charms all their own. The number system with base eleven is known as the unodecimal number system. It is also a positional number system that has two parts the Integral part or integers and the fractional part or fractions, set a part by a radix point. Powers of 12 are used for the positional values of a number. For example (A38.1B5)_{12}.

The following expression shows the position and the power of the base 12 [2]:

\[\text{...12}^1 \text{12}^2 \text{12}^3 \text{12}^4 \text{12}^{-1} \text{12}^{-2} \text{12}^{-3} \text{...}\]

The arithmetic operations such as addition, subtraction, multiplication and division of decimal numbers can be also performed on duodecimal numbers. The number twelve, a highly composite number, is the smallest number with four non-trivial factors (2, 3, 4, 6), and the smallest to include as factors all four numbers (1 to 4) within the subitizing range. The five most elementary fractions (½, ⅓, ⅔, ¼ and ¾) all have a short terminating representation in duodecimal (0.6, 0.4, 0.8, 0.3 and 0.9, respectively), so it is a more convenient number system for computing fractions than other number systems, such as the binary, octal, decimal and hexadecimal systems.

C. Tridecimal Number System

The number system with base thirteen is known as the tridecimal number system. In this system thirteen symbols are used to represent numbers and these are numerals 0 through 9 and alphabets A, B and C. This is an alphanumeric number system because its uses both alphabets and numerical to represent a unodecimal number. It is also a positional number system that each bit position corresponds to a power of 13. It has two parts the integral part or integers and the fractional part or fractions, set a part by radix point. For example (AC8.5B3)_{13}.

In tridecimal number system the leftmost bit is known as most significant bit (MSB) and the right most bit is known as least significant bit (LSB). The following expression shows the position and the power of the base 13:

\[\text{...13}^1 \text{13}^2 \text{13}^3 \text{13}^4 \text{13}^5 \text{13}^6 \text{13}^{-1} \text{13}^{-2} \text{13}^{-3} \text{...}\]

The arithmetic operations like addition, subtraction, multiplication and division operations of decimal numbers can be also performed on tridecimal numbers. It appears in several science fiction stories: Carl Sagan's novel Contact references a message hidden inside pi that is most striking in base 11, as that permits it to be displayed in binary code. In the television series Babylon 5, the Minbari use base-11 mathematics, according to the show's creator. The unodecimal number system is also used to check digit for ISBN.

D. Quadrodecimal Number System

The number system with base fourteen is known as the quadrodecimal number system. It requires fourteen symbols. Since there are only ten common decimal digits, the notation can be extended by using letters A, B, C and D to represent values 10, 11, 12 and 13 respectively. It is also positional system and has two parts the Integral part or integers and the fractional part or fractions, set a part by a radix point. For example (4DA.C04)_{14}.
The following expression shows the position and the power of the base 14 [2]:

\[14^414^314^214^114^0.14^114^214^3\ldots\]

The arithmetic operations such as addition, subtraction, multiplication and division of decimal numbers can be also performed on quadrodecimal numbers. This system is infrequently used. It finds applications in mathematics as well as fields such as programming for the HP 9100/A/B calculator, image processing applications and other specialized uses.

E. Pentadecimal Number System

The number system with base fifteen is known as the pentadecimal number system. It requires fifteen symbols. Since there are only ten common decimal digits, the notation can be extended by using letters A, B, C, D and E to represent values 10, 11, 12, 13, 14 and 15 respectively. This is an alphanumeric number system because its uses both alphabets and numerical to represent a pentadecimal number. It is also a positional number system that each bit position corresponds to a power of 15. It has two parts the Integral part or integers and the fractional part or fractions, set a part by radix point. For example (AE8.DB3)15

The following expression shows the position and the power of the base 15:

\[15^115^0.15^115^215^3\ldots\]

The arithmetic operations like addition, subtraction, multiplication and division operations of decimal numbers can be also performed on pentadecimal numbers. This system is infrequently used. It finds applications in mathematics as well as fields such as telephony routing over IP and other specialized uses.

F. Heptadecimal Number System

The number system with base seventeen is known as the heptadecimal number system. It requires seventeen symbols. Since there are only ten common decimal digits, the notation can be extended by using letters A, B, C, D, E, F, G, and H to represent values 10, 11, 12, 13, 14, 15, 16 and 17 respectively. This is also an alphanumeric number system because its uses both alphabets and numerical to represent a heptadecimal number. It is also a positional number system that each bit position corresponds to a power of 17. It has two parts the Integral part or integers and the fractional part or fractions, set a part by radix point. For example (GE8.DF3)17

The following expression shows the position and the power of the base 17:

\[17^117^0.17^117^217^3\ldots\]

The arithmetic operations like addition, subtraction, multiplication and division operations of decimal numbers can be also performed on heptadecimal. This number system is very important for many scientific applications, as well as for engineering and other practical uses. Numbers in base 17 can be tough, but are kind and soft if treated appropriately.

G. Octodecimal Number System

The number system with base eighteen is known as the octodecimal number system. It requires eighteen symbols. Since there are only ten common decimal digits, the notation can be extended by using letters A, B, C, D, E, F, G and H to represent values 10, 11, 12, 13, 14, 15, 16, 17, 18 respectively. It is also positional system and has two parts the integral part or integers and the fractional part or fractions, set a part by a radix point. For example (H6A.G04)18

The following expression shows the position and the power of the base 18:

\[18^18^0.18^18^218^3\ldots\]

The arithmetic operations such as addition, subtraction, multiplication and division of decimal numbers can be also performed on octodecimal numbers.

H. Nonadecimal Number System

The number system with base nineteen is known as the nonadecimal number system. It requires nineteen symbols. Since there are only ten common decimal digits, the notation can be extended by using letters A, B, C, D, E, F, G, H, I and J to represent values 10, 11, 12, 13, 14, 15, 16, 17, 18, and 19 respectively. It is also positional system and has two parts the integral part or integers and the fractional part or fractions, set a part by a radix point. For example (H6I.G04)19

The following expression shows the position and the power of the base 19:

\[19^19^0.19^19^219^3\ldots\]

The arithmetic operations such as addition, subtraction, multiplication and division of decimal numbers can be also performed on nonadecimal numbers.

I. Vigesimal Number System

The number system with base twenty is known as the vigesimal number system. It requires twenty symbols. Since there are only ten common decimal digits, the notation can be extended by using letters A, B, C, D, E, F, G, H, I and J to represent values 10, 11, 12, 13, 14, 15, 16, 17, 18, 19 respectively. It is also positional system and has two parts the integral part or integers and the fractional part or fractions, set a part by a radix point. For example (J7A.C13)20

The following expression shows the position and the power of the base 20 [2]:

\[20^120^0.20^120^220^3\ldots\]

The arithmetic operations such as addition, subtraction, multiplication and division of decimal numbers can be also performed on vigesimal numbers. This system is widely used nearly all over the world in various languages. This system is unique to our current decimal system, which has a base 10, in that the Mayan's used a vigesimal system, which had a base 20. The use of 20 as a grouping (or base) number was used by many cultures throughout our human history (most likely)
VI. ARITHMETIC OPERATION AND INTER-CONVERSIONS OF STRANGE NUMBER SYSTEM

Arithmetic operations play an important role in various digital systems such as computers, process controllers, signal processors, computer graphics, and image processing. Arithmetic is at the heart of the digital computer, and the majority of arithmetic performed by computers is binary arithmetic, that is, arithmetic on base two numbers. The arithmetic operations such as addition, subtraction, multiplication, and division on strange numbers also play a significant role in computing. Arithmetic operations are performed in a different way in digital computers, depending upon the manner in which the underlying numbers are represented in the computer. In particular, digital arithmetic operations come in two principal varieties: integer operations and floating-point operations. Integer operations are performed on integer numbers, or on numbers stored in a variant of integer number representation known as fixed-point representation. Floating-point operations are performed upon numbers stored in the computer in floating-point representation. Table II shows some basic arithmetic operations performed on strange numbers. The essentials of decimal arithmetic operations have been drilled into us so that we do addition and subtraction almost by instinct. We do binary arithmetic, as well as that of other numbering systems, in the same way that we do decimal arithmetic. The only difference is that we have more digits to use.

### TABLE II: ARITHMETIC OPERATIONS OF STRANGE NUMBER SYSTEM

#### 1) Addition and Subtraction

<table>
<thead>
<tr>
<th>Number System</th>
<th>Addition</th>
<th>Subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unodecimal</td>
<td>(2A)₁₁ + (04)₁₁ = (33)₁₁</td>
<td>(2A)₁₁ - (04)₁₁ = (26)₁₁</td>
</tr>
<tr>
<td>Duodecimal</td>
<td>(2B)₁₂ + (05)₁₂ = (34)₁₂</td>
<td>(2B)₁₂ - (05)₁₂ = (26)₁₂</td>
</tr>
<tr>
<td>Tridecimal</td>
<td>(1C)₁₃ + (05)₁₃ = (2₄)₁₃</td>
<td>(1C)₁₃ - (05)₁₃ = (1₇)₁₃</td>
</tr>
<tr>
<td>Quadrodecimal</td>
<td>(1D)₁₄ + (0₉)₁₄ = (2₃)₁₄</td>
<td>(1D)₁₄ - (0₉)₁₄ = (1₉)₁₄</td>
</tr>
<tr>
<td>Pentadecimal</td>
<td>(1E)₁₅ + (0₂)₁₅ = (5₅)₁₅</td>
<td>(1E)₁₅ - (0₂)₁₅ = (1₇)₁₅</td>
</tr>
<tr>
<td>Heptadecimal</td>
<td>(1G)₁₇ + (0₃)₁₇ = (5E)₁₇</td>
<td>(1G)₁₇ - (0₃)₁₇ = (1₁)₁₇</td>
</tr>
<tr>
<td>Octodecimal</td>
<td>(1H)₁₈ + (0₅)₁₈ = (9D)₁₈</td>
<td>(1H)₁₈ - (0₅)₁₈ = (0₇)₁₈</td>
</tr>
<tr>
<td>Nonadecimal</td>
<td>(2J)₁₉ + (0₇)₁₉ = (1C)₁₉</td>
<td>(2J)₁₉ - (0₇)₁₉ = (0₈)₁₉</td>
</tr>
<tr>
<td>Vigesimal</td>
<td>(1J)₂₀ + (0₃)₂₀ = (5H)₂₀</td>
<td>(1J)₂₀ - (0₃)₂₀ = (0D)₂₀</td>
</tr>
</tbody>
</table>

This section describes the conversion of one base to another. Radix Divide and Multiply Method is generally used for conversion. There is a general procedure for the operation of converting a decimal number to a number in base r. If the number includes a radix point, it is necessary to separate the number into an integer part and a fraction part, since each part must be converted differently.

One way to convert a decimal number to its binary equivalent is the sum-of-weights method. Here, we determine the set of weights (which are in powers of base r) whose sum is the number in question. We observe that multiplying a number x by r^n is equivalent to shifting left the number representation of x by n positions, with zeroes appended on the right. To convert a decimal integer to a specific base, we divide the number successively by the base of this number until the quotient becomes zero. The remainders form the answer, with the first remainder serving as the least significant bit (LSB) and the last remainder the most significant bit (MSB).

To convert a decimal fraction to a specific base, we multiply the number successively by the base of this number, removing the carry in each step, until the fractional product is zero or until the desired number of bits is collected. The carries form the answer, with the first carry serving as the MSB and the last as the LSB.

In the converting values, especially fractional values – between bases, there might be instances when the values are to be corrected within a specific number of places. This may be done by truncation or rounding. In truncation, we simply chop a portion off from the fraction. In rounding, we need to examine the leading digit of the portion we intend to remove.

Apart from the techniques for conversion between related bases such as those discussed above, for general conversion between two bases, the approach is to convert the given value first into decimal, followed by converting the decimal value into the target base. When performing these conversions, it
becomes important for all to indicate the base that the number is written in, as it will become very easy to confuse how the number is being represented. Table III shows the inter-conversions between various numbers.

TABLE III: BASE INTER-CONVERSION OF STRANGE NUMBER SYSTEM

<table>
<thead>
<tr>
<th>Step-1</th>
<th>Part-A</th>
<th>Decimal to any base [unodecimal, … , vigesimal]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integer:</td>
<td>repeated division by base</td>
<td></td>
</tr>
<tr>
<td>Fraction:</td>
<td>repeated multiplication by base</td>
<td></td>
</tr>
<tr>
<td>Part-B</td>
<td>Any base [unodecimal, … , vigesimal] to decimal</td>
<td></td>
</tr>
<tr>
<td>Integer:</td>
<td>sum of [(+ve weights) × (integer)]</td>
<td></td>
</tr>
<tr>
<td>Fraction:</td>
<td>sum of [(-ve weights) × (fraction)]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step-2</th>
<th>(→)1, 12, 13, 14, 15, … to (→)10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct conversion not applicable</td>
<td></td>
</tr>
<tr>
<td>First (→)11, 12, 13, 14, 15, ... to (→)10 Then (→)10 to (→)11, 12, 13, 14, 15, ...</td>
<td></td>
</tr>
</tbody>
</table>

VII. CONCLUSION

Here we have elaborated the concepts of strange number system (SNS) and proposed an easy, short and simple approach to fulfill the needs, number representation, arithmetic operations and inter conversion with different bases, represented in tabulated form used in the digital world specially computer science and technology. As we have seen that, not only traditional numbers are used in digital world, but there are some strange numbers, which are also very common and frequently used in most of the digital technologies and devices. Due to the benefits of strange number representation, which include greater speed of arithmetic operations realization, greater density of memorized information, better usage of transmission paths and decreasing of pin number of integrated circuits, this paper concludes that strange number system even though they are not yet more commercially available, remain a viable field for research, and have a promising future as a replacement for traditional number system. This study will be very helpful for researchers and knowledge seekers to easy understanding and practicing of number systems as well as to memories them for those who are in the field of computer science and technology.

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