ISSN 2047-3338

Vague Logic in Approximate Reasoning

Supriya Raheja, Reena Dadhich and Smita Rajpal

Abstract— **Approximate Reasoning using fuzzy logic provides a realistic framework for human reasoning. The concept of vague logic introduced by Gau & Buehrer [3] is the higher order fuzzy logic. Our present work is based on the concept of vague Logic. In this paper we are defining the approximate reasoning implication rules Generalized Modus Ponens (GMP) and Generalized Modus Tollens (GMT) using vague logic. As a special case we are also reducing the concept of GMT and GMP using fuzzy logic with the help of example.**

Index Terms— **Vague Logic, Linguistic Variable, Approximate Reasoning (AR), GMP and GMT**

I. INTRODUCTION

IN the real word, natural language and human knowledge have a big concern of imprecision and vagueness. The theory of fuzzy logic given by Zadeh [9] in 1965 is a good mathematical and methodological basis for capturing the uncertainties, imprecise information associated with human related processes, such as identifying relationships, thinking and reasoning. There are number of generalized forms [1]-[2]-[3]-[6] of fuzzy set theory [9] exists in the literature like fuzzy theory, two-fold fuzzy theory, vague theory, intuitionistic fuzzy theory, probabilistic fuzzy theory, etc. Our present work is based on the concept of vague theory recently introduced by Gau and Buehrer [3]. Vague sets are the higher order fuzzy sets. This generalization makes the procedure more complex but the result achieved by higher order theory could be the better one.

Let X is a universe of discourse; say the collection of all students of ITM University. Let A be a vague set of all "good-in-computers students" of the universe X, and B be a fuzzy set of all "good-in-computers students" of X. Suppose that an intelligent agent A1 proposes the membership value $\mu_B()$ for the element x in the fuzzy set B by his best intellectual capability. On the contrary, another intelligent agent A2 proposes independently two membership values $t_A(x)$ and $f_A(x)$ for the same element in the vague set A by his best intellectual capability. The amount $t_A(x)$ is the true-membership value of x and $f_A(x)$ is the false-membership value of x in the vague set A. Both A1 and A2 being human agents have their limitation of perception, judgment, processing-capability with real life complex situations. In the case of fuzzy set B, the agent A1 proposes the membership value $\mu_B(x)$ and proceeds to his next computation. There is no higher order check for this membership value in general. In the second case, the agent A2 proposes independently the membership values $t_A(x)$ and $f_A(x)$, and makes a check by using the constraint, $t_A(x) + f_A(x) \leq 1$. If it is not honored, the agent has a chance to rethink and to reshuffle his judgment procedure either on 'evidence against' or on 'evidence for' or on both.

Supriya Raheja is with the ITM University, Gurgaon, India (e-mail: supriya.raheja@ gmail.com)

Dr. Smita Rajpal, is with ITM University, Gurgaon, India (e-mail: smita_rajpal@yahoo.co.in)

Dr. Reena Dadhich is with the Govt. Engineering college, Ajmer, India (e-mail: reena.dadhich@gmail.com)

In 1979 Zadeh [5], father of fuzzy logic introduced the theory of approximate reasoning. This theory provides a powerful framework for reasoning in the face of imprecise and uncertain information. Prof. Zadeh says, "In its narrow sense, fuzzy logic is logic of approximate reasoning which may be viewed as a generalization and extension of multivalued logic". The main motivation of the theory of approximate reasoning is the desire to create a qualitative framework that will use to derive an approximate conclusion from imprecise knowledge [7].

Central to this theory is the concept of linguistic variable that reflects that most of the human reasoning is approximate rather than exact. A linguistic variable is a variable whose values are expressed in words rather than numbers. For example, Age is a linguistic variable if its values are linguistic rather than numerical, i.e., young, not young, very young, quite young, old, not very old and not very young, etc., rather than 20, 21, 22, 23.... Zadeh introduced a number of translation rules which allow us to represent some common linguistic statements in terms of propositions in our natural language.

In fuzzy logic and approximate reasoning, Fuzzy inference rules are basis for approximate reasoning suggested by Zadeh [5]. Fuzzy sets & fuzzy relations are used to represent simple and complex fuzzy propositions in fuzzy logic. Rules of inference are used to derive new propositions from a given collection of fuzzy propositions. Zadeh introduced the form of inference rule in approximate reasoning. The most important fuzzy implication inference rules are the Generalized Modus Ponens (GMP) and the Generalized Modus Tollens (GMT). The classical Modus ponens rule says if p is true and $p \rightarrow q$ is true then q is true.

The more prevalent fuzzy logic is one in which rules of inference such as Modus Ponens, Modus Tollens and Hypothetical syllogism are "fuzzified". The fuzzy implication inference (GMP or GMT) is based on the compositional rule of inference for approximate reasoning. The compositional rule of inference is a tool to implement the generalized "Modus Ponens" which allows us to treat a key problem in Approximate Reasoning [12].

We have a piece of expert's knowledge presented by means of a rule if x is A then y is B where A and B are the linguistic variables. A new fact is considered with the form x is A', A' being another linguistic term on X. We must infer a conclusion i.e. the consequent of the rule, which will have the form y is B', B' will be obviously another vague statement. The approximate reasoning process is modeled by means of an implication function f: [0, l] x [0, l] -> [0, 1]. Thus expert's knowledge is captured in a fuzzy relation R(x, y) = f (A(x), B(y)). All data are combined into the statement:

if x is A then y is B and x is A'.

B' is finally computed by B'(Y) = sup min (A'(X), R(X, Y))

The classical Modus Tollens rule says if p → q is true and q is false then p is false. The Generalized Modus Tollens, if the rule is "if x is A then y is B" and the fact is "y is B□" [11], the consequent will be x is A□ and A□ can be computed by

A□ (x) = sup main (B□ (y), R(X, Y)

In this paper, we are introducing the approximate reasoning with vague logic.

II. Preliminaries

Next Section discusses the basic preliminaries of vague set theory. Then we discuss the literature review of fuzzy logic for approximate reasoning.

Over the classical set theory, Zadeh introduced the concept of fuzzy set theory which has been applied almost all the fields such as computer sciences, medical sciences, to solve the mathematical problems, expert systems and many more.

Let X = {u_1, u_2... u_n} be the universe of discourse. The membership function μ_A (u) of a fuzzy set A is a function $\mu_A : X \rightarrow [0,1]$. A fuzzy set A in X is defined as the set of ordered pairs A = {(u, $\mu_A(u)$) : u∈ X }, where $\mu_A(u)$ is the grade of membership of element u in the set A [9]. The greater $\mu_A(u)$, the greater "the element u belongs to the set A". With this concept Prof. Zadeh introduced fuzzy set theory.

A. Vague Set

Gau and Buehrer [3] pointed out that single membership value μ_A (u) combines the 'evidence for u' and the 'evidence against u'. It does not indicate how much there is of each. Consequently, there is a necessity of higher order fuzzy sets like vague sets, which could be treated as a further generalization of Zadeh's fuzzy sets [9].

Definition 1: A vague set (or in short VS) A in the universe of discourse X is characterized by two membership functions given by [4]:

1) A truth membership function t_A: X ∈ [0, 1],

2) A false membership function f_A: X ∈ [0, 1],

where $t_A(u)$ is a lower bound of the grade of membership of u derived from the 'evidence for u', and $f_A(u)$ is a lower bound on the negation of u derived from the 'evidence against u', and their total amount cannot exceed 1, i.e., $t_A(u) + f_A(u) \leq 1$. Thus the grade of membership of u in the vague set A is bounded by a subinterval [t_A (u), 1- f_A (u)] of [0, 1]. This indicates that if the actual grade of membership is μ(u), then $t_A(u) \leq (u) \leq 1 - f_A(u)$.

The vague set A is written as A = {< u, [t_A (u), f_A (u)] >: u ∈ X, where the interval [t_A (u), 1 - f_A (u)] is called the 'vague value' of u in A and is denoted by V_A (u). For example, consider a universe X = {young, old, very old}. A vague set A of X could be A = {<young, [.8, .2]>, <old, [.3, .5], <very old, [.4, .6]>}. Here in case of linguissic variable "young" t_A is .8 and 1-f_A is 2.

Definition 2: The complement of a vague set A is denoted by A□ and is defined by

$$t_{A^{□}}(x) = f_A(x)$$
$$1 - f_{A^{□}}(x) = 1 - t_A(x)$$

B. Vague Relations and Composition

In this paper, our work is based on the theory of vague relations. In this section we discuss the recent literature on vague relations & their properties.

VAGUE RELATION (VR)

Let X and Y are two universes. A vague relation (VR) denoted by R (X→ Y) of the universe X with the universe Y is VS of the Cartesian product X × Y. The true membership value t_R(x, y) estimates the strength of the existence of the relation of R-type of the object x with the object y, whereas the false membership value f_R(x, y) estimates the strength of the non-existence of the relation of R-type of the object x with the object y [8]-[10]. Generally the relation R (X → Y) is denoted by the notation R.

Example: Consider two universes X = {a, b} and Y = {x, y, z}. Let R be a vague relation of the universe X with the universe Y (X→Y) proposed by an intelligent agent as shown by the following table.

Table 1
VR: R (X→Y)

R (X→Y)	X	Y	Z
A	(.6,.3)	(.3,.5)	(.7,.3)
B	(.2,.5)	(.7,.3)	(.4,.4)

The proposed VR reveals the strength of vague relation of every pair X →Y; For example, it reveals that the object 'a' of the universe X has R-relation with the element 'z' of Y with the following estimation:

Strength of existence of the relation = .7

Strength of non-existence of the relation = .3

A relation E (X→Y) is called a Complete Relation from the universe X to the universe Y if V_E(x, y) = [1, 1], ∀ (x, y) ∈ X×Y. A relation Φ (X→Y) is called a Null Relation from the universe X to the universe Y if V_Φ(x, y) = [0, 0], ∀ (x, y) ∈ X×Y.

COMPOSITION OF VRs

The composition of binary relations is a concept of forming a new relation S ° R from two given relations R and S. A vague set and a vague relation could also form a new vague relation with a useful significance. Similarly two vague relations, under a suitable composition, could too yield a new vague relation. Composition of a relation is important for application, because of the reason that if a relation of a universe X with another universe Y is know and if a relation of the universe Y with a third universe Z is known then the relation of X with Z could be computed. In our paper, we use composition of a vague set and a vague relation to yield a new relation.

Definition 3 (Composition of a VS and a VR): Let A be a VS of the universe X and R be a VR of the universe X with another Let A be a VS of the universe X and R be a VR of the universe X with another universe Y. The composition of R with A, denoted by B = R °A, is a VS in Y given by

$$V_{RoA}(y) = [isup_{x \in X} \{t_A(x) \wedge t_R(x, y)\},\ isup_{x \in X} \{(1 - f_A)(x) \wedge (1 - f_R)(x, y)\}].$$

Definition 4 (Composition of two VRs): Let R(X → Y) and S(Y → Z) are two VRs. Then the composition relation B = R ° S is a VR of X with Z given by

$$V_{RoS}(x, z) = [isup_{y \in Y} \{t_R(x, y) \wedge t_S(y, z)\},\ isup_{y \in Y} \{(1 - f_R)(x, y) \wedge (1 - f_S)(y, z)\}].$$

This composition yields a vague-valued link between the objects x (of X) and z (of Z) through the elements y (of Y). Clearly R ° S ≠ S ° R.

C. Fuzzy Logic and AR

Suppose that we are given an $x' \in X$ and want to find an $y' \in Y$ which corresponds to x' under the rule-base.

R1 : If x = x1 then y = y1
R2 : If x = x2 then y = y2
. . . . . .
Rn : If x = xn then y = yn
fact: $x = x'$
consequence: $y = y'$

Let x and y be linguistic variables, e.g. "x is high" and "y is small". The basic problem of approximate reasoning is to find the membership function of the consequence part. For this Prof. Zadeh has introduced a large number of translation rules.

In fuzzy logic and approximate reasoning, the most important fuzzy implication inference rule is the Generalized Modus Ponens (GMP). Fuzzy inference rules are the basis of approximate reasoning suggested by Zadeh [5]-[7].

Definition 5: The Generalized Modus Ponens rule says

Rule : if x is A then y is B
fact : x is A'
consequence: y is B'

where the consequence B' in the matrix form is determined as a composition of the fact and the fuzzy implication operator.

$B' = A' \circ (A \rightarrow B)$ that is equivalent to

$$B'(y) = \sup_{x \in X} \min \{A'(x), (X \rightarrow Y)(x, y)\}, y \in Y.$$

Definition 6: The Generalized Modus Tollens inference rule says:

Rule : if x is A then y is B
fact : y is B'
consequence: x is A'

$A' = B' \circ (A \rightarrow B)$ that is equivalent to

$$A'(x) = \sup_{y \in Y} \min \{B'(y), (A \rightarrow B)(x, y)\}, x \in X.$$

III. Related Work

Prof. L. A. Zadeh [5] described the term linguistic variable, a variable whose values are words or sentences in a natural or artificial language. He also characterized linguistic variable by a quintuple (χ , T (χ), U, G, M). Author discussed the concept of hedges such as very, quite, extremely, etc., as well as the connectives and and or in this paper. The author discussed the concept of a linguistic variable provides a means of approximate characterization of phenomena which are too complex or too ill-defined to be amenable to description in conventional quantitative terms which leads to fuzzy logic. By providing a basis for approximate reasoning, that is, a mode of reasoning which is not exact nor very inexact, such logic may offer a more realistic framework for human reasoning than the traditional two-valued logic. The author described the main applications of the linguistic approach that lie in the realm of humanistic systems-especially in the fields of artificial intelligence, linguistics, human decision processes, pattern recognition, psychology, law, medical diagnosis, information retrieval, economics and related areas.

E. H. Mamdani described an application of fuzzy logic in designing controllers for industrial plants [11]. The discussed method has been applied to pilot scale plants as well as in a practical industrial situation. The author also discussed the potential for using fuzzy logic in modeling and decision making. From the application point of view both the learning situation described as well as decision making can be best framed in terms of hierarchical structures. The work described in this paper demonstrates the great usefulness of applying Approximate Reasoning using fuzzy logic to management and other humanistic systems.

IV. Implementation of Approximate Reasoning Using Vague Logic

In the previous section, we discussed the literature review of fuzzy logic and approximate reasoning. In this section we introduce the vague logic with approximate reasoning by applying the vague set theory over the GMP and GMT. As Gau & Buehrer defined the major advantage of vague sets over fuzzy sets is that vague sets disjoin the positive and negative evidence for membership of an element in the set. We not only have an estimate of how likely it is that an element is in the set, but we also have a lower and upper bound on this likelihood. This lower/upper bound can be used to perform constraint propagation. For instance, constraint propagation can be used to detect inconsistencies in assignments of intervals to Boolean expressions involving the sets.

A. Definition of GMP for Vague logic

The Generalized Modus Ponens rule in vague logic says

Rule : if x is A then y is B
fact : x is A'
consequence: y is B'

Let x and y be the linguistic variables and here these linguistic variables are defined by the member functions that provides the information "evidence for u" and "evidence against u" i.e. t_A and f_A. Equation (1) shows how the consequence B'

can be determined as a composition of fact in vague logic and vague implication operator.

$$V_{B'} = A' \circ (VR) \text{ that is equivalent to}$$

$$V_{B'} = [\text{isup}_{x \in X} \{t_{A'}(x) \wedge t_R(x, y)\},\ \text{isup}_{x \in X} \{(1 - f_{A'})(x) \wedge (1 - f_R)(x, y)\}] \quad (1)$$

Where $t_R(x, y) = J[t_A(x), t_B(y)] = \min[1, 1 - t_A(x) + t_B(y)]$ &

$f_R(x, y) = J[f_A(x), f_B(y)] = \min[1, 1 - f_A(x) + f_B(y)]$

B. Definition of GMT for Vague logic

The Generalized Modus Tollens rule for vague logic says

Rule : if x is A then y is B

Fact : y is B′

consequence: x is A′

Equation (2) shows how the consequence A′ can be determined as a composition of fact in vague logic and vague implication operator:

$$V_{A'} = B' \circ (VR) \text{ that is equivalent to}$$

$$V_{A'} = [\text{isup}_{x \in X} (t_{B'}(x) \wedge t_R(x, y)\},\ \text{isup}_{x \in X}\{(1 - f_{B'})(x) \wedge (1 - f_R)(x, y)\}] \quad (2)$$

Where $t_R(x, y) = J[t_A(x), t_B(y)] = \min[1, 1 - t_A(x) + t_B(y)]$ &

$f_R(x, y) = J[f_A(x), f_B(y)] = \min[1, 1 - f_A(x) + f_B(y)]$

V. NUMERICAL ANALYSIS

Case1: Let X = {x1, x2, x3} and Y = {y1, y2} be the sets of values of variables X, Y.

Let A = [0.4, 0.6]/x1 + [0.8, 0.9]/x2 + [0.5, 0.6]/x3 and

B = [0.9, 0.9]/y1 + [0.3, 0.5]/y2.

Let A′ = [0.5, 0.6]/x1 + [0.8, 0.9]/x2 + [0.6, 0.7]/x3.

According to the equation (1) defined in previous section.

$$V_{R(x,y)} = \begin{array}{c|cc} & y1 & y2 \\ \hline x1 & [1,1] & [.9,.9] \\ x2 & [1,1] & [.5,.7] \\ x3 & [1,1] & [.8,.9] \end{array}$$

$$V_{B'} = \begin{array}{c|c} x1 & [.5,.6] \\ x2 & [.8,.9] \\ x3 & [.6,.7] \end{array} \circ \begin{array}{c|cc} & y1 & y2 \\ \hline x1 & [1,1] & [.9,.9] \\ x2 & [1,1] & [.5,.7] \\ x3 & [1,1] & [.8,.9] \end{array}$$

$$= \begin{array}{c|cc} & y1 & y2 \\ \hline x1 & [.5,.6] & [.5,.6] \\ x2 & [.8,.9] & [.5,.7] \\ x3 & [.6,.7] & [.6,.7] \end{array}$$

$V_{B'}$ = [0.8, 0.9]/y1 + [0.6, 0.7]/y2

Case2: Let X = {x1, x2, x3} and Y = {y1, y2} be the sets of values of variables X, Y.

Let A = [0.4, 0.6]/x1 + [0.8, 0.9]/x2 + [0.5, 0.6]/x3 and

B = [0.9, 0.9]/y1 + [0.3, 0.5]/y2.

Let B′ = [0.8, 0.8]/y1 + [0.6, 0.7]/y2.

According to equation (2):

$$V_{R(x,y)} = \begin{array}{c|cc} & y1 & y2 \\ \hline x1 & [1,1] & [.9,.9] \\ x2 & [1,1] & [.5,.7] \\ x3 & [1,1] & [.8,.9] \end{array}$$

$$V_{A'} = \begin{array}{cc} y1 & y2 \\ \hline [.8,.8] & [.6,.7] \end{array} \circ \begin{array}{c|cc} & y1 & y2 \\ \hline x1 & [1,1] & [.9,.9] \\ x2 & [1,1] & [.5,.7] \\ x3 & [1,1] & [.8,.9] \end{array}$$

$$= \begin{array}{c|cc} & y1 & y2 \\ \hline x1 & [.8,.8] & [.6,.7] \\ x2 & [.8,.8] & [.5,.7] \\ x3 & [.8,.8] & [.6,.7] \end{array}$$

$V_{A'}$ = [0.8, 0.8]/x1 + [0.8, 0.8]/x2 + [0.8, 0.8]/x3

VI. CONCLUSION

In this paper we have implemented approximate reasoning. We have used the vague set tool in order to get the better approximate result as it separates the positive and negative evidence of membership in the relation. We have extended the Generalized Modus Ponens & Generalized Modus Tollens implication rules with the vague logic. As a special case, the method reduces to a method of approximate reasoning using fuzzy logic. In this work we considered an example to illustrate the GMP and GMT implication rules in matrix form. Obviously, if there is no in deterministic element in the membership values throughout the computations, the notion of vague relations reduces to the notion of fuzzy relation.

ACKNOWLEDGMENT

Author would like to thank Dr. Reena Dadhich who provided invaluable comments in. Author would like to thank Dr. Smita Rajpal for her inputs in the conceptualization of the paper and her support throughout for technical discussions.

REFERENCES

[1] K. Atanassov, "Intuitionistic fuzzy sets", Fuzzy Sets and Systems, 20:87– 96, 1986.

[2] K. Atanassov, "Intuitionistic Fuzzy Sets: Theory and Applications", Physica-Verlag, New York, 2000.

[3] W. L. Gau and D. J. Buehrer, "Vague sets", IEEE Transaction on systems, Man and Cybernetics, 23:610– 614, 1993.

[4] V. Novak, I. Perfilieva, J. Mochkorg, "Mathematical principles of vague logic", (In Russian) FIZMATLIT, Moskow, (2006) 352 .

[5] L.A. Zadeh, "The concept of linguistic variable and its application to approximate reasoning", American Elsevier Publishing Company (1973).

[6] Bustince H. and Burillo P., "Vague sets are intuitionistic fuzzy sets",

Fuzzy Sets and Systems 79 (1996) 403-405.

[7] Christer Carlsson, "Uncertainty modeling and Approximate reasoning", IEEE, 1060-3425/98, 1998.

[8] H. Khan, M. Ahmd, and R. Biswas, "Vague relations", Inter. Jour. Of Computational Cognition, VOL. 5, No. 1, S 1542-5908(07), 2007

[9] L. A. Zadeh, "Fuzzy sets", Information and Control, 8:338–356, 1956

[10] Smita Rajpal, M.N. Doja, Ranjit Biswas," A Method of Vague Search to Answer Queries in Relational Databases", Information: International Journal of Japan, Vol.10, No.6, November, Japan, 2007.

[11] E. H. Mamdani, "Application of fuzzy logic to approximate reasoning using linguistic synthesis", IEEE Transactions, Volume: C-26, Issue: 12, 1182 - 1191, Dec. 1977.

[12] Banibrata Mondal, Dipankar Mazumdar and Swapan Raha, "Similarity in Approximate Reasoning", International Journal of Computational Cognition", VOL. 4, No. 3, Sept. 2006.

Supriya Raheja, ITM University, pursuing her PhD in Computer Science from Banasthali University. She had done her engineering from Hindu college of Engineering, Sonepat and masters from Guru Jambeshwar University of Science and Technology, Hisar. She is working as a Reviewer/Committee member of various International Journals and Conferences. Her total Research publications are nine.

Dr. Reena Dadhich is presently working as an Associate Professor and Head of the Department of Master of Computer Applications at Engineering College Ajmer, India. She received her Ph.D. (Computer Sc.) and M.Sc. (Computer Sc.) degree from Banasthali University, India. Her research interests are Algorithm Analysis & Design, Wireless Ad-Hoc Networks and Software Testing. She has more than 12 years of teaching experience. She is working as an Editorial Board Member / Reviewer/Committee member of various International Journals and Conferences. She has written many research papers and books.

Dr. Smita Rajpal, ITM University, completed her PhD in Computer Engineering. She has a total work experience of 11 years. She is specialized in TOC, Compiler Design, Soft Computing and RDBMS. She is a Java certified professional. She is working as an Editorial Board Member / Reviewer/Committee member of various International Journals and Conferences. She is an active member of IEEE. Her biography is a part of Marquis who's who in the world, 2010.Her total Research publications are 20 and book chapter's-5.She has published three books.