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# Basis for Symmetric Matrix Data Gravity Interpretation

<sup>1</sup>Adewole O.O., and <sup>2</sup>Ewumi T.O.

<sup>1</sup>Department of Physics, University of Ibadan, Ibadan, Nigeria

<sup>2</sup>Department of Physics, University of Ado-Ekiti, Ekiti, Nigeria

**Abstract**– Gravity method has tremendously pervaded the field of geophysical exploration and thus discussed in this work. The basis for symmetric matrix data gravity interpretation is proposed. A symmetric matrix representation is discussed and optimistically forms a veritable tool for gravity data interpretation. The fundamental law of gravitational attraction is essential in delineating gravity data interpretation and the gravity effect or anomaly representation is evidently expressed by a finite difference matrix in the discussion. A computational extension can be done based on the symmetric matrix representation with available gravity data.

**Index Terms**– Symmetric Matrix, Gravity Data, Gravity Effect and Gravitational Attraction

## I. INTRODUCTION

Gravity method has been a veritable tool in geophysical exploration for several years. The gravity method is a non-destructible geophysical technique that measures differences in the earth's gravitational field at specific locations. In engineering and environmental sciences, it has found numerous applications among locating voids and karst features, buried stream valleys, water table levels and the determination of soil layer thickness. The gravity method depends on variations produced in the measured gravitational field produced by different earth materials of different bulk densities i.e., mass. These variations can be interpreted by a variety of analytical and computer methods to determine the depth, geometry and density that causes the field variations.

The interpretation of Bourguer gravity anomalies ranges from just manually inspecting the grid or profiles for variations in the gravity field to more complex methods that involve separating the gravity anomaly due to an object of interest from some sort of regional gravity field. Several manual and computer techniques including graphical smoothing and polynomial surface fitting are applied in performing these techniques.

The gravity method can be a relatively easy technique to perform and interpreted. It requires only simple but precise data for processing. The technique has good depth penetration when compared to ground penetrating radar, high frequencies electromagnetic and D.C resistivity techniques and is not affected by the high conductivity values of near surface clay rich soils. Additionally, lateral boundaries of sub surface

features can be easily obtained especially through the measurement of the derivatives of the gravitational field.

## II. THEORY

The gravity method involves variation in acceleration due to gravity and thus thye Newton's law of universal gravitation becomes extremely important. The acceleration due to gravity,  $g$  can be obtained from Newton's law of motion; where  $m$  is a test mass in equation (1)b, hence the Newton's law of universal gravitation is obtained where  $M_e$  is the mass of the earth.

$$g = GMm_e r' / R_e^2 \dots\dots\dots (1). a$$

$$F = ma \dots\dots\dots (1). b$$

The units for  $g$  are  $\text{cm/s}^2$  in c.g.s system and are commonly known as Gals, where the average acceleration due to gravity at the earth's surface is 980Gals. Variations in gravity from  $10^{-1}$  to  $10^{-3}$  are considered in most gravity studies and milliGal (mGal) is used by most workers. Even some detailed engineering and environmental work involve microGal ( $\mu\text{Gal}$ ) variations.

Since the gravity method is concerned with determining sub-surface variations in mass distributions, most interpretation techniques involve the solution of the previous equation (1) due to some mass distribution. This can be accomplished by solving for the gravity field due to a generalized mass distribution using an integral equation. In most gravity applications, recent work on the application of the gravity gradient tensor to exploration problem (Mickus et al, 2002, Mikhailov et al, 2007) could involve all three components of the gravity field.

*Delineating the contour for the integral:*

$$\Delta g = \int_s K(x, x_0, z_0) \rho(x, x_0) dz_0 dx_0 \dots\dots (2)$$

Where  $\Delta g(x)$  is the gravity effect of a two dimensional mass distribution bounded by the surface,  $s, x_0$  and  $z_0$  are the source coordinates,  $K(x, x_0, z_0)$  is the kernel giving the gravity effect of an elementary block of unit density, and  $\rho(x_0, z_0)$  is the actual density of the block.

In equation (2), it is tacitly assumed that the density is a continuous function of space co-ordinates which renders the evaluation of the integral (2) very difficult. Thus, to perform numerical computations, a discrete variation of density with position is assumed. The integral (2) is then reduced to finite difference matrix:

$$\Delta g = \sum_{i=1}^m \sum_{j=1}^n k_{ij} \rho_{ij} \dots \dots \dots (3)$$

Which in matrix notation could be expressed viz:

$$|\Delta| = |k||\rho| \dots \dots \dots (4)$$

Where the body consists of m rows and n columns. The value of m is fixed through successive approximation to give a close fit between the calculated and observed gravity values. The limit on n is set by the memory space of the computer.

### III. FURTHER DISCUSSION

Density is a very important parameter in gravity interpretation of data. The sub-surface variations of mass is very important and requires that the density of the material of interest or the density contrast between the material of interest and the surrounding material be known. The density can be determined by various techniques. The best technique is to acquire rock samples within the study area and determine their average density. The density logs obtained from drill logs can also be used but this is not always available. Density can also be estimated from experimental relationships relating compression seismic velocities obtained from seismic refraction surveys and density (Nafe and Drake, 1957, Birch, 1961). Also, average density values from tables obtained from numerous measurements of rocks, soil and mineral samples (Johnson and Olhoeft, 1984, Telford et al, 1990).

The regional-residual anomaly can be accomplished by many techniques. The simplest technique is the manual method such as the graphical smoothing where a simple smooth regional anomaly subtracted from the observed gravity anomaly to obtain a residual anomaly. The manual technique has the advantage of giving information on the lateral location of the source bodies which can be used select a correct regional anomaly.

Most other regional-residual anomaly separation techniques involve mathematical operations using a computer. The major limitation of the mathematical method is that they do not accurately represent the true residual gravity anomaly due to a specific body. Thus, they should not be used for quantitative interpretation of the sub-surface but only for qualitative interpretation (Ulyrch, 1968). The most common mathematical techniques are surface fitting and weighted averaging. Surface fitting involves a least-square fitting of 2-D polynomial (Beltrac et al,19991, Kim et al, 2008) or 2-D Fourier series (James, 1968) of different orders to the original gridded Bouguer gravity data to represent a regional gravity anomaly map.

The higher the surface order, the greater the fit to the original data, however, high-order surfaces are usually not desired, as they contain part of the anomaly that is desired.

The kernel in equation (2) is the effect of a point due to an elementary two-dimensional rectangular block of unit density and is given by the formula (Heiland, 1946).

$$K_{ij} = 2y \{ x'_{ij} \ln(z_{ij}^2 + x_{ij}^2 / z_{ij}^2 + x_{i,i}^2)^{1/2} - (x_{ij} - WS) \ln(z_{ij}^2 + (x_{ij} - WS)^2 / z_{ij}^2 + (x_{ii} - WS)^2)^{1/2} + z_{ij} [\tan^{-1} x_{ij} / z_{ij} - \tan^{-1} (x_{ii} - WS) / z_{ij}] - z_{ij-1} [\tan^{-1} / z_{ij-1} - \tan^{-1} (x_{ij} - WS) / z_{ij-1}] \}$$

Where y is the universal gravitation constant, z<sub>ij</sub> is the z co-ordinate of the far horizontal face, x<sub>ij</sub> is the x-co-ordinate of the far vertical face of j-th block reckoned from the observation point as origin, and WS is the width of the block.

### IV. CONCLUSION

Interpretation of gravity data is extremely important in geophysical exploration and related applications, thus it is expedient to have a robust tool for gravity data interpretation. The fundamental basis of symmetric matrix has been elaborated and hopefully would be a veritable analytical tool in gravity data interpretation. A numerical extension can be made and appropriate data can be thoroughly treated.

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